Exam 2

Tuesday, March 23rd.

Show your work for full credit.

1. (20 points) Find the eigenvalues and associated eigenvectors for the matrix:

$$\begin{bmatrix} 2 & 9 \\ 4 & 2 \end{bmatrix}$$

Answer: Eigenvalues are -4, 8. Corresponding normalized eigenvectors are $[3, -2]$, $[\frac{3}{5}, \frac{2}{5}]$.

2. Suppose a population of insects has two stages. Stage 1 individuals produce on average two individuals offspring, and one quarter of stage 1 individuals survive to become stage 2 individuals. In stage 2, individuals produce on average five individual offspring, and stage two individuals die after one period in stage two. Note that all offspring are stage one individuals.

   (a) (6 points) Suppose there were 80 individuals in stage one and 40 individuals in stage two at time 0 . How many individuals would there be in each stage two periods later?
   Answer: 820 in stage 1, 90 in stage 2.

   (b) (7 points) If the population were to exist for a long time with the above mentioned conditions staying the same, what would the long–term growth rate of the population be per time period? If at time 100, the population of insects has reached the equilibrium growth rate with 1 million individuals present, approximately how many individuals would be present at time 101?
   Answer: long term growth rate=dominant eigenvalue of the Leslie matrix=$\frac{5}{2}$. Number of individuals present in time 101 is 2.5 million.

   (c) (7 points) If the population were to exist for a long time with the above mentioned conditions staying the same, approximately what fraction of the population would be in each of the two stages?
   Answer: Equilibrium population structure=normalized eigenvector corresponding to the dominan teigenvalue=$[\frac{10}{11}, \frac{1}{11}] \simeq$ [90.9%, 9.1%] .

3. (20 points) Use the following matrices to perform each of the following computations, if possible. If not possible, briefly explain why not.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ -2 & 0 \end{bmatrix}$$

   (a) $2AC$ not defined

   (b) $A + CB$ not defined

   (c) $BC - A = \begin{bmatrix} -2 & 0 \\ 1 & 6 \end{bmatrix}$

4. (20 points) Suppose crop is sprayed with long-lasting pesticide which either remains in the crop or goes to the soil. Suppose each week 20% of the pesticide in the crop is transfered to soil and 30% of pesticide is transferred to the crop.

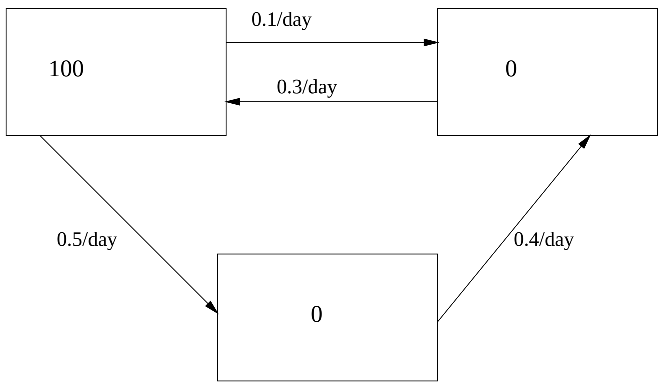
   (a) Give a matrix model for this system.
   Answer: $\begin{bmatrix} C_{n+1} \\ S_{n+1} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix} \begin{bmatrix} C_n \\ S_n \end{bmatrix}$

(b) Find the equilibrium state for this system. Answer: $\begin{bmatrix} 40 \\ 60 \end{bmatrix}$ if the total amount is 100.

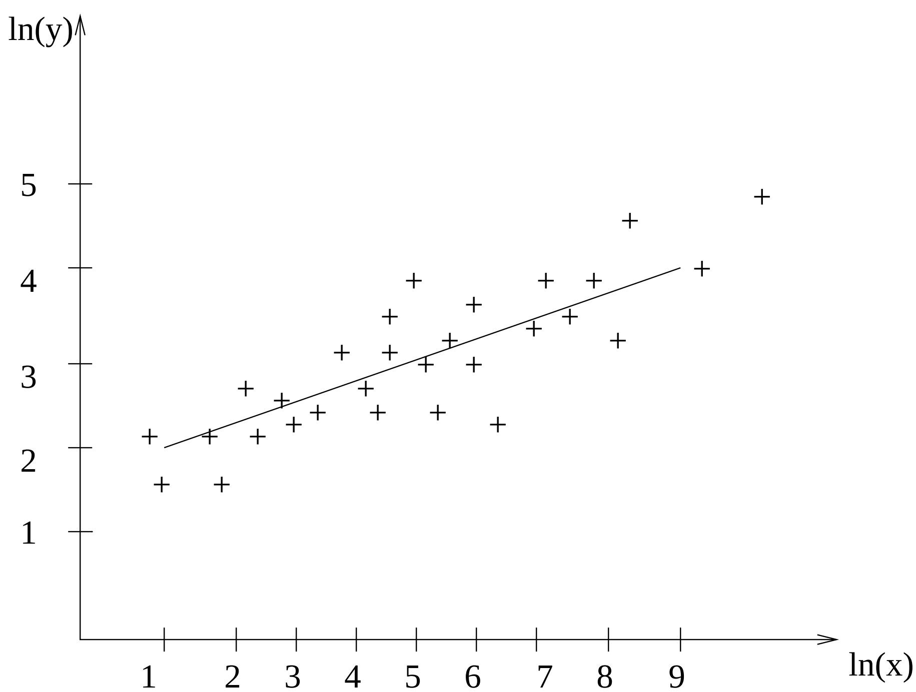
5. (20 points) In the compartmental diagram shown below the transfer coefficients are shown next to the arrows and the contents at the initial time are given in the boxes.

   (a) Find the transfer matrix for this system. Answer: $\begin{bmatrix} 0.4 & 0.3 & 0 \\ 0.1 & 0.7 & 0.4 \\ 0.5 & 0 & 0.6 \end{bmatrix}$

   (b) Find the state of the system 2 days later after. Answer: $\begin{bmatrix} 19 \\ 31 \\ 50 \end{bmatrix}$



Extra Credit (5 points) Find the equation of the line and then the original equation.



Answer: Equation of the line (may vary a little): $Y = 0.25X + 1.25$, where $Y = \ln(y)$, $X = \ln(x)$
Original equation: $y = 3.49x^{0.25}$.