

Real Analysis Lecture Notes

Stefan Richter

November 23, 2011

Contents

1	σ-algebras and Measures.	1
1.1	σ -algebras	1
1.2	The extended real numbers	5
1.3	Measures	5
1.4	Outer measures	8
1.5	Construction of outer measures	12
1.6	Some concluding thoughts on the process that got us Lebesgue measure.	20
1.7	Exercises	22
2	Measurable functions	26
2.1	Topological preliminaries	26
2.2	Real and complex-valued measurable functions	28
2.3	Measurable functions with values in the extended reals	30
2.4	Measurable simple functions	33
2.5	Exercises	35
3	Integrals	37
3.1	Integrals of nonnegative functions	37
3.2	Integrals of complex valued functions	44
3.3	Sets of measure zero	46
3.4	Integration Theorems for complex-valued functions	49
3.5	Exercises	51
4	Measure and Topology	54
4.1	Urysohn's Lemma in \mathbb{R}^n	54
4.2	Radon measures in \mathbb{R}^n and Lusin's Theorem	57
4.3	Exercises	60

5	L^p spaces	63
5.1	Convexity and inequalities	63
5.2	Banach spaces and $L^p(\mu)$	68
5.3	Density of $C_c(W)$ in $L^p(\mu)$, $1 \leq p < \infty$	74
5.4	Exercises	75
6	Elementary Hilbert space Theory	78
6.1	Closest elements, orthogonality, and projections	78
6.2	Linear functionals and the Riesz representation theorem	82
6.3	Exercises	86
7	Complex Measures	87
7.1	The total variation measure	87
7.2	The structure of complex measures	90
7.3	The Lebesgue-Radon-Nikodym Theorem	96
7.4	Linear Functionals on $L^p(\mu)$, $1 \leq p < \infty$	101
7.5	Exercises	106
8	Product Spaces	107
8.1	Product σ -algebras and the Monotone Class Theorem	108
8.2	Product Measures and Fubini's Theorem	110
8.3	Completion of product measure spaces and Lebesgue measure on \mathbb{R}^n	117
8.4	Convolutions	118
8.5	Exercises	120