

Math 545, HW # 11

due 11-11-11

1. Let \mathcal{H} be a Hilbert space. A set $S \subseteq \mathcal{H}$ is called orthonormal, if $\|x\| = 1$ for all $x \in S$ and if $x \perp y$ for all $x, y \in S$ with $x \neq y$.

Let $S = \{x_1, \dots, x_n\}$ be orthonormal in \mathcal{H} . Set

$$\mathcal{M} = \left\{ \sum_{i=1}^n \alpha_i x_i : \alpha_i \in \mathbb{C} \right\}.$$

(a) Show that \mathcal{M} is a closed subspace of \mathcal{H} .

(b) If $x \in \mathcal{H}$, then show that $P_{\mathcal{M}}x = \sum_{i=1}^n \langle x, x_i \rangle x_i$.

Problems **2,3**: Chapter 5 #9, 11