MATH 472 MIDTERM EXAM

March 12, 2019

(1) Consider the linear system $Ax = b$, where

$$A = \begin{pmatrix} 1.297 & 0.8648 \\ 0.2161 & 0.1441 \end{pmatrix}, \qquad b = \begin{pmatrix} 0.8644 \\ 0.1440 \end{pmatrix}$$

Solve the system using Gauss elimination. Do all calculations using 4 decimal digit rounding arithmetic. Explain what happens.

(2) Show that the matrix A

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$

is symmetric positive definite. Find its Cholesky decomposition.

(3) Find the LDL^T factorization of the matrix

$$A = \begin{pmatrix} 2 & -2 & 2 \\ -2 & 0 & -6 \\ 2 & -6 & -2 \end{pmatrix}.$$

Is this matrix positive definite? Justify your answer.

(4) Prove that an orthogonal triangular matrix is diagonal.

(5) Find the QR factorization of the matrix

$$A = \begin{pmatrix} 0 & 0 & 6 \\ 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \end{pmatrix}$$

using Householder transformations.

(6) Apply the Doolittle direct factorization technique to the matrix

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 6 & -2 \\ 4 & -3 & 8 \end{pmatrix}.$$

(7) Suppose A and B are both symmetric positive definite matrices. State whether the following statement are true or false. Give a simple proof or provide a counterexample

(a) $A + B$ is positive definite
(b) $A - B$ is positive definite
(c) A^T is positive definite
(d) A^3 is positive definite

(8) Show that a Householder matrix $H(\mathbf{v})$ is not positive definite unless $\mathbf{v} = 0$.

(9) Consider the overdetermined system $A\mathbf{x} = \mathbf{b}$, with

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \\ -2 & 4 \end{bmatrix}, \quad \text{and } \mathbf{b} = \begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix}.$$

(a) Use the method of normal equations to find the Least-Squares solution.
(b) Find the QR factorization of A and use it to compute the Least-Squares solution.