Quiz 1 Tue Jan 22

$$[A\,|\,b] = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right]$$

(a) $\xrightarrow[-3\times r_1 \text{ add to } r_3]{1\times r_1 \text{ add to } r_2}$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{array}\right]$ $\xrightarrow{-10\times r_2 \text{ add to } r_3}$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & -52 & -104 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & -52 & -104 \end{array}\right]$ $\xrightarrow[-\frac{1}{52}\times r_3]{-1\times r_2}$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{array}\right]$ ref

(c) $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{array}\right]$ $\xrightarrow[-2\times r_3 \text{ add to } r_1]{5\times r_3 \text{ add to } r_2}$ $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$

$\downarrow -1\times r_2 \text{ add to } r_1$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$ rref

(d) Already from (a) we see that system is consistent since # of nonzero rows in $\tilde{A}$ = # of nonzero rows of $[\tilde{A}\,|\,\tilde{b}] = 3$

(e) x_1, x_2, x_3 are pivot variables since columns 1, 2, 3 of $\tilde{A}$ contain a leading one.

using the rref, we solve to find $x_1 = 3, x_2 = 1, x_3 = 2$ i.e. a unique solution. This is consistent with the fact that there are no free variables.