Math 251
Sample Test 2

1. Determine whether the set $\{x^2+76x+23, 13x^2-29x+15, -5x^2+x+28\}$ is linearly independent or not.

2. Determine whether the set $\{x^2 + 76x + 23, 13x^2 - 29x + 15, -5x^2 + x + 28, x^2 - 3x + 5\}$ is linearly independent or not.

3. If A is an $m \times n$ matrix, show that $\text{rank}(A) \leq \min\{m, n\}$.

4. Suppose A is a 7×4 matrix. Only one of the following statements is correct. Identify it with justification

   (a) $\text{rank}(A)$ can be any integer between 0 and 7.

   (b) $\text{nullity}(A)$ cannot be less that 3.

   (c) The system $A\mathbf{x} = \mathbf{0}$ cannot have any nonzero solutions.

   (d) the $\text{nullity}(A)$ is at most 4.

5. Let A and B be two matrices such that BA can be formed.

   (a) Show that the row space of BA is a subspace of the row space of A, in other words every linear combination of rows of BA is a linear combination of rows of A.

   (b) Show that if in addition B is square and invertible, then the row space of A is a subspace of the row space of BA.

   In conclusion, if B is invertible then the row spaces of BA and A are identical.

6. Let A and B be two matrices such that BA can be formed.

   (a) Show that $\ker(A)$ is a subspace of $\ker(BA)$. For this, show that if $A\mathbf{x} = \mathbf{0}$, then $BA\mathbf{x} = \mathbf{0}$.

   (b) Show that if in addition B is square and invertible, then $\ker(BA)$ is a subspace of $\ker(A)$.

   In conclusion, if B is invertible then $\ker(BA) = \ker(A)$.

7. Let

$$A = \begin{bmatrix} 1 & 2 & -1 & -2 \\ -3 & 1 & 3 & 4 \\ -3 & 8 & 4 & 2 \end{bmatrix}.$$

   Find bases for each of the following subspaces

   (a) The row space of A,

   (b) the column space of A,

   (c) $\ker(A)$,

   (d) $\ker(A^T)$.

8. The vectors $(1, -2, -1), (1, 4, 8), (0, 2, 3), (1, 2, 5)$ are obviously linearly dependent(why?). Identify a subset of linearly indepedent vectors and express the others as linear combinations of these.