Math 251
Sample Midterm

The test will consist essentially of two parts: **Computation** and **Theory**. I will not give here specific examples since you can identify examples from the text. If you know the computational procedures and the relevant theory then you should be able to complete the tasks. Here is a sample, but necessarily exhaustive, list of each

Computation

1. Matrix operations: multiplication, addition, scalar multiplication.
2. Solve a linear system using Gauss Elimination. Identify the free and pivot variables. Write down the general solution in terms of free parameters (if any).
3. Solve a system using Cramer's rule.
4. Compute determinants using cofactor expansion or triangularization.
5. Compute the inverse of a matrix using Gauss-Jordan elimination.
6. Compute the inverse of a matrix from the adjoint.

Theory This requires good knowledge of and ability to use the theory that was developed in class and not necessarily the proofs. One excellent way of preparing for this is to look at the True-False questions in the text. Here's a very small sample. In case the statement is false, it would be helpful to see how you could change the statement to make it true. For instance, for 4) a true statement would be : If A has a row of zeros, then all but one column of $adj(A)$ must be zero.

1. True or False: If AB and BA are both well defined, then A and B must be square.
2. True or False: The system with augmented matrix $[A|b] = \begin{bmatrix} 1 & a_{12} & a_{13} & b_1 \\ 0 & 0 & a_{23} & 0 \\ 0 & 0 & 1 & b_3 \end{bmatrix}$ is always consistent independently of the values of $a_{12}, a_{13}, a_{23}, b_1, b_3$.
3. True or False: The augmented matrix of a linear system is 4×7. Then, it is not possible for this system to have only one solution.
4. True or False: If a matrix A has a row of zeros then so does $adj(A)$.
5. True or False: If the system $A\mathbf{x} = \mathbf{b}$ has a solution then the system $A\mathbf{x} = \mathbf{c}$ has a solution.
6. True or False: If A is square and the system $A\mathbf{x} = \mathbf{0}$ has multiple solutions the $\det(A) = 0$.
7. True or False: If a matrix B is obtained from A by interchanging rows 1 and 2 and then interchanging rows 2 and 3, then $det(B) = det(A)$.
8. Show that A is invertible if and only if AA^T is invertible.
9. Suppose B is invertible. Show that $AB = BA$ if and only if $AB^{-1} = B^{-1}A$.