General Information:

- The exam will be held on Monday April 20, 2009. It will be closed-book and closed-notes.
- The exam will cover Sections 6.1-6.6, 8.1,8.2 from the textbook and the supplemental notes on the spring-mass system.
- All assigned homework problems and problems that are similar in nature are fair game for the exam.

You should be able to do the following:

- Set up a differential equation discribing the dynamics of a spring/mass system. The general spring/mass system is described by the second order differential equation

$$mx''(t) + \beta x'(t) + kx(t) = f(t)$$

where m, β and k are constants representing the mass of an object attached to the spring, the damping coefficient and the spring constant respectively. $f(t)$ is the driving force of the system.

- know how to determine whether the system is under, critically or over-damped from the corresponding characterstics of the differential equation.
- Need to revise how to use the **method of undetermined coefficients** (section 4.4) to find a particular solution of the constant coecient linear dierential equation

$$ay'' + by' + cy = f(t)$$

where the forcing function f(t) is a linear combination of functions of the form $t^k e^{at} \cos bt$ and $t^k e^{at} \sin bt$ for various choices of the constants a, b, c, and d, and non-negative integer k.

- Know how to solve the **Cauchy-Euler equation**

$$at^2 y'' + bty' + cy = 0$$

where a, b, and c are real constants by means of the roots r_1, r_2 of the characterstics equation

$$q(r) = ar^2 + (b - a)r + c = 0.$$

- Know how to use the **Reduction of Order** method to obtain a second independent solution of a homogeneous second order linear differential equation, assuming a solution $y_1(t)$ is known.

- If $\{y_1(t), y_2(t)\}$ is a fundamental solution set for the homogeneous equation

  $$y'' + b(t)y' + c(t)y = 0$$

  ; know how to use the **method of variation of parameters** to find a particular solution y_p of the non-homogeneous equation

  $$y'' + b(t)y' + c(t)y = f(t).$$

  Variation of Parameters: Find y_p in the form $y_p = u_1y_1 + u_2y_2$ where u_1 and u_2 are unknown functions whose derivatives satisfy the following two equations:

  $$\begin{aligned} u_1'y_1 + u_2'y_2 &= 0 \\ u_1'y_1' + u_2'y_2' &= f(t) : \end{aligned}$$

  Solve the above system for u_1' and u_2', and then integrate to find u_1 and u_2.

- Know what it means for a function to have a jump discontinuity and to be piecewise continuous.

- Know how to piece together solutions on different intervals to produce a solution of one of the initial value problems

  $$y' + ay = f(t); \quad y(t_0) = y_0$$

  or

  $$y'' + ay' + by = f(t) \quad y(t_0) = y_0; y'(t_0) = y_1;$$

  where $f(t)$ is a piecewise continuous function on an interval containing t_0.

- Know what the unit step function (also called the Heaviside function) (h(t - c)) and the on-off switches $(\chi_{[a,b)})$ are:

  $$h(t-c) = h_c(t) = \begin{cases} 0 & \text{if } 0 \le t < c \\ 1 & \text{if } t \ge c \end{cases}$$

  and,

  $$\begin{aligned} \chi_{[a,b)} &= \begin{cases} 1 & \text{if } a \le t < b; \\ 0 & \text{otherwise} \end{cases} \\ &= h(t-a) - h(t-b) \end{aligned}$$

  and know how to use these two functions to rewrite a piecewise continuous function in a manner which is convenient for computation of Laplace transforms.