Math 231 Quiz 1 SU17 Name(s)

No Work-No Credit

Due by Thursday, 6/08/2017, not accepted late!

Use Picard's Method to find the approximation, $\phi_3(x)$, of the solution to:

$\frac{dy}{dx} = 2 - \frac{y}{x}, \; y(1) = 2$ Let $\phi_0(x) = 2$

Then $\phi_1(x) = 2 + \int_1^x \left(2 - \frac{2}{t}\right) dt = 2 + (2t - 2\ln t)\Big|_1^x$

$= 2 + \left[(2x - 2\ln x) - (2 - 2\ln(1))\right]$

$\phi_1(x) = 2x - 2\ln x$

$\phi_2(x) = 2 + \int_1^x \left(2 - \frac{2t - 2\ln t}{t}\right) dt = 2 + \int_1^x \left(2 - 2 + 2\frac{\ln t}{t}\right) dt$

$= 2 + 2\int_1^x \frac{\ln t}{t}\, dt = 2 + \left((\ln t)^2\right)_1^x = 2 + \left[(\ln x)^2 - (\ln 1)^2\right]$

$\phi_2(x) = 2 + (\ln x)^2$

$\phi_3(x) = 2 + \int_1^x \left(2 - \frac{2 + (\ln t)^2}{t}\right) dt = 2 + \int_1^x \left(2 - \frac{2}{t} - \frac{(\ln t)^2}{t}\right) dt$

$= 2 + \left(2t - 2\ln t - \tfrac{1}{3}(\ln t)^3\right)_1^x =$

$2 + \left[\left(2x - 2\ln x - \tfrac{1}{3}(\ln x)^3\right) - \left(2 - 2\ln 1 - \tfrac{1}{3}(\ln 1)^3\right)\right]$

$\phi_3(x) = 2x - 2\ln x - \tfrac{1}{3}(\ln x)^3.$