Exam 4 Math 142 Summer 99 Name
No Work-No Credit SS#

1) Determine whether this sequence converges or diverges. If convergent, determine the limit.

$\{e^{-n} \ln n\}_{n=2}^{\infty}$

Let $f(x) = \frac{\ln x}{e^x}$. Then we have $\lim_{x \to \infty} \frac{\ln x}{e^x} \overset{LR}{=} \lim_{x \to \infty} \frac{1}{x e^x} = 0$

Therefore the sequence converges to zero.

2) Use the Integral Test to determine if the series converges or diverges.

$\sum_{n=2}^{\infty} n 2^{-n^2}$. We have $\int_2^{\infty} x 2^{-x^2} \, dx = \lim_{A \to \infty} \int_2^{A} x 2^{-x^2} \, dx$

Now integrate and then evaluate the limit. Then $\frac{-1}{2 \ln 2} \lim_{x \to \infty} \left(2^{-x^2}\right)_2^A =$

$\frac{-1}{2 \ln 2} \lim_{x \to \infty} \left(\frac{1}{2^{A^2}} - \frac{1}{2^4} \right) = \frac{1}{32 \ln 2}$. So the series is convergent since the improper integral is convergent.

3) Determine if the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} (-1)^n \frac{(n+1)^2}{n^5+1}$$

a) First check for absolute convergence. Ratio Test gives 1, so no help. Consider $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)^2}{n^5+1}$, try the Limit Comparison Test with Σ 1/n3, a convergent p-series.

Consider, $\displaystyle \lim_{n \to \infty} \frac{(n+1)^2 / (n^5+1)}{1/n^3} = \lim_{n \to \infty} \frac{(n+1)^2 n^3}{n^5+1} = 1 > 0$ so both either converge or diverge. Since the test series converges the original series converges.

Thus the series is **absolutely** convergent.

4) Determine whether the given series converge or diverge(must state reasons)

a) $\sum_{n=1}^{\infty} \frac{2^n}{n^3+1}$

Ratio test: $$\lim_{n \to \infty} \left| \frac{2^{n+1}}{2^n} \bullet \frac{n^3+1}{(n+1)^3+1} \right| = 2 > 1$$

So divergent.

b) $\sum_{n=1}^{\infty} \frac{1}{1+\ln n}$

Since $\frac{1}{1+n} < \frac{1}{1+\ln n}$

And $\sum_{n=1}^{\infty} \frac{1}{1+n}$ diverges, so
The given series diverges.

c) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\tan^{-1} n}$

$$\lim_{n \to \infty} \frac{\sqrt{n}}{\tan^{-1} n} = \frac{2}{p} \lim_{n \to \infty} \sqrt{n} = \infty \neq 0$$

So divergent by the test for divergence.

d) $\sum_{n=0}^{\infty} \frac{1}{2+\left(\frac{1}{2}\right)^n}$

$$\lim_{n \to \infty} \frac{1}{2+\left(\frac{1}{2}\right)^n} = \frac{1}{2} \neq 0$$

So divergent by the test for
Divergence.

5) Determine the radius and interval of convergence for:

$$\sum_{n=1}^{\infty} (-1)^n \frac{4^{2n}}{\sqrt{n+1}} (x-3)^n$$

Using the Ratio Test: $|x-3| \lim_{n \to \infty} \left| \frac{16^{n+1}}{16^n} \bullet \frac{\sqrt{n+1}}{\sqrt{n+2}} \right| = 16|x-3| < 1$ for absolute convergence. Then | x-3 | < 1/16. So the Radius of convergence is R = 1/16.

Now determine the Interval of convergence.

Let x = 47/16. This gives $\sum \frac{1}{\sqrt{n+1}}$ which is divergent (this series is just a divergent p-series).

Let x = 49/16. This gives $\sum (-1)^n \frac{1}{\sqrt{n+1}}$ which is convergent by AST.

So I = $\left(\frac{47}{16}, \frac{49}{16} \right]$.

5) For $f(x) = \sqrt{1+x}$, determine $T_3(x)$ (c = 0), where $T_3(x) = \sum_{i=0}^{3} \frac{f^{i}(0)}{i!} x^{i}$.

$f(x) = (1+x)^{1/2}$ $f(0) = 1$

$f'(x) = \frac{1}{2}(1+x)^{-1/2}$ $f'(0) = \frac{1}{2}$

$f''(x) = -1/4(1+x)^{-3/2}$ $f''(0) = -1/4$

$f'''(x) = 3/8(1+x)^{-5/2}$ $f'''(0) = 3/8$

then f(x) $\approx 1 + \frac{1/2}{1!} x + \frac{-1/4}{2!} x^2 + \frac{3/8}{3!} x^3$

So $\sqrt{1+x} \approx 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$.