Math 142 Exam 2 Spring 2003 Name

No Work-No Credit 3/26/03 SS#

For all problems follow the directions carefully. Use the entire display for values when using the calculator. Make sure you set up each integral.

1) Find the centroid of the region determined $y = x^2$, $y = 4$.

By symmetry, $\bar{x} = 0$. Then we have $\bar{y} = \dfrac{M_x}{m} = \dfrac{\frac{1}{2}\int_{-2}^{2}\left(16 - x^4\right)dx}{\int_{-2}^{2}\left(4 - x^2\right)dx} = \dfrac{128/5}{32/3} = \dfrac{12}{5}$.

2) Determine the length of arc of one loop of the curve with parametric equations $x = t^2$, $y = t^3 - 3t$. (One loop is generated by $-\sqrt{3} \le t \le \sqrt{3}$)

L = $\int ds = \int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{4t^2 + \left(3t^2 - 3\right)^2}\, dt \cong 10.7350606837$

3) Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \dfrac{2}{x}$, $x = 4$, $x = 6$ and $y = 0$ about the line $x = 9$.

Using shells:

$V = \int_4^6 2p\left(9 - x\right)\left(\dfrac{2}{x}\right)dx \cong 20.7242821484$.

4) Find the area of the region below the curve $r = \frac{q}{2}$ and above the extended polar axis for $0 \le q \le p$.

A = $\frac{1}{2}\int_0^p \left(\frac{q}{2}\right)^2 dq = \frac{p^3}{12}$

5) Use Simpson's Rule to approximate $\int_0^2 2^x \, dx$, with n = 4. You must complete the table for credit. Use values rounded to 4 decimal places!!

i	x_i	$f(x_i)$	M	M* $f(x_i)$
0	0	1	1	1
1	0.5	1.41421	4	5.65685
2	1	2	2	4
3	1.5	2.82843	4	11.31371
4	2	4	1	4

$\int_0^2 2^x \, dx \cong \frac{1}{6}(25.97056) \cong 4.32843$

6) Suppose that 3 N of force is needed to stretch a spring from its natural length of 22 cm to a length of 30 cm. How much work is needed to stretch it from 25 cm to 35 cm?

3 = k(0.08) so k = 37.5. Then $W = \int_{0.03}^{0.13} 37.5\,x \, dx = 0.3\,J$