Math 142 Exam 2 Spring 06 Name

No Work-No Credit Last 4 Digits

1) Use the trapezoidal rule to approximate $\int_0^2 5^x\,dx$. Let n = 4 and round all decimal values to five (5) decimal places. Must complete the table for credit!

| i | $x_i$ | $f(x_i)$ | M | $M*f(x_i)$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 |
| 1 | 0.5 | 2.23607 | 2 | 4.47214 |
| 2 | 1 | 5 | 2 | 10 |
| 3 | 1.5 | 11.18034 | 2 | 22.36068 |
| 4 | 2 | 25 | 1 | 25 |

$\Delta x =$ 0.5

$\sum M * f(x_i) =$ 62.83282

$\int_0^2 5^x\,dx \approx$ 0.25(62.83282) = 15.70820

2) Determine the partial fraction decomposition for: $\dfrac{3x^2+x+4}{x\,(x+2)(x-2)}$.

$$\frac{3x^2+x+4}{x\,(x+2)(x-2)} = \frac{A}{x}+\frac{B}{x+2}+\frac{C}{x-2} = \frac{-1}{x} + \frac{7/4}{x+2} + \frac{9/4}{x-2}$$

3) Evaluate: $\int \sin^{-1} x \, dx$

Let u = $\sin^{-1}x$ dV = dx

du = $\frac{1}{\sqrt{1-x^2}}$ dx V = x then $\int \sin^{-1} x \, dx = x\sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx$

Now, $\int \frac{x}{\sqrt{1-x^2}} dx = -\frac{1}{2}\int \alpha^{-1/2} \, d\alpha = -\alpha^{1/2} \quad \Rightarrow -\sqrt{1-x^2}$

Let $\alpha = 1 - x^2$

$d\alpha = -2x\,dx$

$-\frac{1}{2} d\alpha = x\,dx$

So $\int \sin^{-1} x \, dx = x \sin^{-1} x + \sqrt{1-x^2} + C$

4) Compute the value of the integral, or prove it divergent. $\int_1^{\infty} \frac{\ln x}{x} dx$.

$$\int_1^{\infty} \frac{\ln x}{x} dx =$$

$$\lim_{A\to\infty} \int_1^{A} \frac{\ln x}{x} dx = \frac{1}{2} \lim_{A\to\infty} (\ln x)^2 \Big|_1^A = \frac{1}{2} \lim_{A\to\infty} \left[ (\ln A)^2 - (\ln 1)^2 \right] = \frac{1}{2} \lim_{A\to\infty} (\ln A)^2 = \infty$$

So divergent!

$$\int \frac{\ln x}{x} dx = \int u\,du = \frac{u^2}{2} \Rightarrow \frac{(\ln x)^2}{2}$$

Consider $Let \;\; u = \ln x$

$$du = \frac{1}{x} dx$$

5) Evaluate: $\int \frac{x^3}{\sqrt{4+x^2}}\,dx$

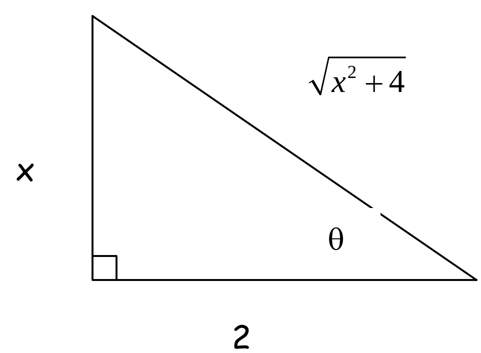
Let x = 2tan $\theta$ $\Rightarrow$ dx = 2sec$^2$ $\theta$ d$\theta$ then the problem becomes

$$\int \frac{x^3}{\sqrt{4+x^2}}\,dx = \int \frac{8\tan^3\theta}{2\sec\theta}\,2\sec^2\theta\,d\theta = 8\int\left(\tan^3\theta\right)\sec\theta\,d\theta = 8\int\left(\sec^2\theta-1\right)\left(\sec\theta\right)\tan\theta\,d\theta =$$

now let u = sec $\theta$ $\Rightarrow$ du = (sec $\theta$)(tan $\theta$) d$\theta$, so we have

$$8\int\left(u^2-1\right)du = 8\left(\frac{u^3}{3}-u\right) \Rightarrow 8\left(\frac{\sec^3\theta}{3}-\sec\theta\right)+C$$

since x = 2tan $\theta$ $\Rightarrow$ $\frac{x}{2} = \tan\theta$




So finally

$$\int \frac{x^3}{\sqrt{4+x^2}}\,dx = 8\left(\frac{\left(x^2+4\right)^{3/2}}{24}-\frac{\sqrt{x^2+4}}{2}\right)+C$$