Exam 4 Math 142 F10 Name Solutions
No work-No credit 9AM Last 4 Digits

1) Determine whether the sequence converges or diverges.
If convergent, find the limit.

$$a_n = \frac{e^n + 2}{e^{2n} - 1}$$

Want $\displaystyle\lim_{n\to\infty} \frac{e^n+2}{e^{2n}-1}\left(\frac{\frac{1}{e^{2n}}}{\frac{1}{e^{2n}}}\right) =$

$$\lim_{n\to\infty} \frac{\frac{1}{e^n} + \frac{2}{e^{2n}}}{1 - \frac{1}{e^{2n}}} = \frac{0}{1} = 0$$

Convergent to 0.

2) Use the Integral Test to determine the convergence or divergence of:

$$\sum_{n=1}^{\infty} \frac{4}{(2+4n)^2}$$

$$4\int_1^{\infty} \frac{1}{(2+4x)^2}\,dx = 4\lim_{T\to\infty}\int_1^T \frac{1}{(2+4x)^2}\,dx = -\lim_{T\to\infty}\left[\cancel{\frac{1}{2+4T}} - \frac{1}{6}\right]$$

$= \frac{1}{6}$ $\therefore$ convergent.

$$\int \frac{1}{(2+4x)^2}\,dx = \frac{1}{4}\int u^{-2}\,du = -\frac{1}{4}\frac{1}{u} + c$$

$$-\frac{1}{4}\frac{1}{2+4x} + c$$

$u = 2+4x$

$du = 4\,dx$

$\frac{1}{4}du = dx$

3) Determine whether the following series is absolutely convergent, conditionally convergent, or divergent.

a) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{4}{2n+1}$

a.) Determine if absolutely convergent, consider $\sum_{n=1}^{\infty} \left| (-1)^{n-1} \frac{4}{2n+1} \right|$

$4 \sum_{n=1}^{\infty} \frac{1}{2n+1}$, Use LCT $\sum \frac{1}{n}$, $\lim_{n \to \infty} \frac{\frac{1}{2n+1}}{\frac{1}{n}} = \lim_{n \to \infty} \frac{n}{2n+1} = \frac{1}{2} > 0$

So not absolutely convergent.

b.) See if conditionally convergent, use AST.

i.) $a_n \geq a_{n+1}$ ✓ since $\frac{1}{2n+1} > \frac{1}{2n+3}$

ii.) $\lim_{n \to \infty} \frac{1}{2n+1} = 0$ ✓

So conditionally convergent.

4) Determine Convergence or Divergence (No work-No credit!!!!)

a) $\sum_{n=1}^{\infty} \frac{3}{n+11}$

LCT $\sum \frac{1}{n}$ div.

$$\lim_{n\to\infty} \frac{\frac{3}{n+11}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{3n}{n+11} = 3 > 0$$

$\therefore$ divergent

b) $\sum_{n=1}^{\infty} \frac{\sqrt{n^3+1}}{5n^3+n^2+2}$

LCT $\sum \frac{1}{n^{\frac{3}{2}}}$ P-series $P = \frac{3}{2} > 1$ conv.

$$\lim_{n\to\infty} \frac{\frac{\sqrt{n^3+1}}{5n^3+n^2+2}}{\frac{1}{n^{\frac{3}{2}}}} = \lim_{n\to\infty} \frac{\sqrt{n^6+n^3}}{5n^3+n^2+2} =$$

$$\lim_{n\to\infty} \frac{\sqrt{n^6+n^3}}{5n^3+n^2+2} \left(\frac{\frac{1}{n^3}}{\frac{1}{n^3}} \right) =$$

$$\lim_{n\to\infty} \frac{\sqrt{1+\frac{1}{n^3}}}{5+\frac{1}{n}+\frac{2}{n^3}} = \frac{1}{5} > 0$$

So convergent.

c) $\sum_{n=1}^{\infty} \frac{4^{n+2}}{7^{n-1}}$

$$7 \cdot 16 \sum_{n=1}^{\infty} \left(\frac{4}{7}\right)^n$$

Geo. $r = \frac{4}{7} < 1$

$\therefore$ convergent

b) $\sum_{n=2}^{\infty} \frac{n \ln n}{2^n}$

Ratio Test $\lim_{n\to\infty} \left| \frac{(n+1)\ln(n+1)}{2^{n+1}} \cdot \frac{2^n}{n \ln n} \right|$

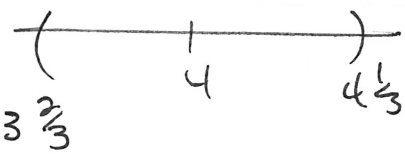
$$\frac{1}{2} \lim_{n\to\infty} \left| \frac{n+1}{n} \cdot \frac{\ln(n+1)}{\ln n} \right| = \frac{1}{2} < 1$$

Convergent.

5) Determine the Radius and Interval of convergence for:

$$\sum_{n=0}^{\infty} \frac{3^n (x-4)^n}{\sqrt{n+3}}$$ use ratio test $$\lim_{n\to\infty}\left|\frac{3^{n+1}(x-4)^{n+1}}{\sqrt{n+4}}\cdot\frac{\sqrt{n+3}}{3^n(x-4)^n}\right| =$$

$$3|x-4| \lim_{n\to\infty}\left|\frac{\sqrt{n+3}}{\sqrt{n+4}}\right| \Rightarrow 3|x-4|<1 \text{ then } |x-4|<\frac{1}{3}=R$$



Check endpoints

Let $x=3\frac{2}{3}$ then $\displaystyle\sum_{n=0}^{\infty}\frac{3^n\left(-\frac{1}{3}\right)^n}{\sqrt{n+3}} = \sum_{n=0}^{\infty}(-1)^n\frac{1}{\sqrt{n+3}}$ not absolutely conv.

since $\sum\frac{1}{\sqrt{n+3}}$ is divergent by LCT $\sum\frac{1}{\sqrt{n}}$ div. p-series

$$\lim_{n\to\infty}\frac{\frac{1}{\sqrt{n+3}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+3}} = 1>0.$$

Try AST for conditional conv. i) $a_n \geq a_{n+1}$ since $\frac{1}{\sqrt{n+3}} > \frac{1}{\sqrt{n+4}}$

ii) $\displaystyle\lim_{n\to\infty}\frac{1}{\sqrt{n+3}} = 0$ so cond. conv.

Let $x=4\frac{1}{3} \Rightarrow \displaystyle\sum_{n=0}^{\infty}\frac{3^n\left(\frac{1}{3}\right)^n}{\sqrt{n+3}} = \sum_{n=0}^{\infty}\frac{1}{\sqrt{n+3}}$ div. by above LCT with $\sum\frac{1}{\sqrt{n}}$.

$$R=\frac{1}{3}, \quad I=\left[3\tfrac{2}{3}, 4\tfrac{1}{3}\right).$$