Exam 4 Math 142 F10 Name Solutions
No work-No credit 11AM Last 4 Digits

1) Determine whether the sequence converges or diverges.
If convergent, find the limit.

$$a_n = \frac{\pi^n}{4^n} = \left(\frac{\pi}{4}\right)^n$$

convergent if $-1 < r \le 1$ & $\frac{\pi}{4} < 1$.

So convergent to $\emptyset$ since

$$\lim_{n\to\infty} \left(\frac{\pi}{4}\right)^n = 0.$$

2) Use the Integral Test to determine the convergence or divergence of:

$$\sum_{n=1}^{\infty} \frac{4}{1+n^2}$$

$$4\int_1^{\infty} \frac{1}{1+x^2}\,dx = 4 \lim_{A\to\infty} \int_1^A \frac{1}{1+x^2}\,dx = 4 \lim_{A\to\infty} \tan^{-1}x \Big|_1^A$$

$$4 \lim_{A\to\infty} \left[\tan^{-1}A - \tan^{-1}1\right] = 4\left[\frac{\pi}{2} - \frac{\pi}{4}\right] = 4\,\frac{\pi}{4} = \pi$$

So convergent

3) Determine whether the following series is absolutely convergent, conditionally convergent, or divergent.

a) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{\sqrt{n(n+1)}}$

a.) Determine if absolutely convergent, consider $\sum_{n=1}^{\infty} \left| (-1)^{n-1} \frac{1}{\sqrt{n(n+1)}} \right|$

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}$ LCT $\sum \frac{1}{n}$, $\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n^2+n}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\sqrt{n^2}}{\sqrt{n^2+n}} =$

$\lim_{n\to\infty} \sqrt{\frac{n^2}{n^2+n}} = 1 > 0$ So Not absolutely conv.

b) Determine if conditionally convergent, use AST.

i.) $a_n \geq a_{n-1}$ since $\frac{1}{\sqrt{n(n+1)}} > \frac{1}{\sqrt{(n+1)(n+2)}}$

ii.) $\lim_{n\to\infty} \frac{1}{\sqrt{n(n+1)}} = 0$.

So conditionally convergent.

4) Determine Convergence or Divergence (No work-No credit!!!!)

a) $\sum_{n=1}^{\infty} \frac{3}{5^n + 11} = 3\sum_{n=1}^{\infty} \frac{1}{5^n+11}$

$5^n+11 > 5^n$ so

$\frac{1}{5^n+11} < \frac{1}{5^n}$ & $\sum_{n=1}^{\infty} (\frac{1}{5})^n$

Geo. $r=\frac{1}{5}<1$

So convergent by comp. to $\sum_{n=1}^{\infty} (\frac{1}{5})^n$.

b) $\sum_{n=1}^{\infty} \frac{\sqrt{n^3+1}}{n^3+5n^2+2}$

Use LCT $\sum \frac{1}{n^{3/2}}$ conv. p-series

$\lim_{n\to\infty} \frac{\frac{\sqrt{n^3+1}}{n^3+5n^2+2}}{\frac{1}{n^{3/2}}} = \lim_{n\to\infty} \frac{\sqrt{n^6+n^3}}{n^3+5n^2+2}$

$= \lim_{n\to\infty} \frac{\sqrt{n^6+n^3}}{n^3+5n^2+2} \cdot \left(\frac{\frac{1}{n^3}}{\frac{1}{n^3}}\right) =$

$\lim_{n\to\infty} \frac{\sqrt{1+\frac{1}{n^3}}}{1+\frac{5}{n}+\frac{2}{n^3}} = 1 > 0$

$\therefore$ convergent

c) $\sum_{n=1}^{\infty} \frac{e^{n+2}}{7^{n-1}}$

$7e^2 \sum_{n=1}^{\infty} (\frac{e}{7})^n$

Geo, $r=\frac{e}{7}<1$

Convergent

b) $\sum_{n=2}^{\infty} \frac{n^2+n+2}{4n^2-3}$

$\lim_{n\to\infty} \frac{n^2+n+2}{4n^2-3} = \frac{1}{4} \neq 0$

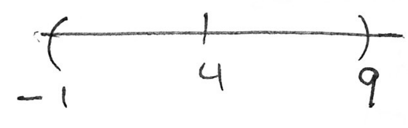
Div. by TFD.

5) Determine the Radius and Interval of convergence for:

$$\sum_{n=0}^{\infty} (-1)^n \frac{(x-4)^n}{5^n\sqrt{n+3}}$$

Use Ratio Test $\lim_{n\to\infty}\left|\frac{(x-4)^{n+1}}{5^{n+1}\sqrt{n+4}} \cdot \frac{5^n\sqrt{n+3}}{(x-4)^n}\right| =$

$\frac{|x-4|}{5} \lim_{n\to\infty}\left|\frac{\sqrt{n+3}}{\sqrt{n+4}}\right|$ so $\frac{|x-4|}{5} < 1 \Rightarrow |x-4| < 5 = R$



check endpoints

at $x=-1 \Rightarrow \sum_{n=0}^{\infty} (-1)^n \frac{(-5)^n}{5^n\sqrt{n+3}} = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n+3}}$ LCT $\sum \frac{1}{\sqrt{n}}$ p-series $p=\frac{1}{2}<1 \therefore$ div.

$\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n+3}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+3}} = 1 > 0$

so divergent.

Let $x=9 \Rightarrow \sum_{n=0}^{\infty} (-1)^n \frac{5^n}{5^n\sqrt{n+3}} = \sum_{n=0}^{\infty} (-1)^n \frac{1}{\sqrt{n+3}}$ Now not absolutely conv. by above.

Try AST for conditional conv.

i) $a_n \geq a_{n+1}$ ✓ since $\frac{1}{\sqrt{n+3}} > \frac{1}{\sqrt{n+4}}$

ii) $\lim_{n\to\infty} \frac{1}{\sqrt{n+3}} = 0$ ✓ cond. conv.

$R=5$ $I = (-1, 9]$.