Math 141 Fall 1998 Exam 4 Name
No Work-No Credit SS#

1) Use Newton's method to approximate the nearest root of $f(x) = \frac{1}{x^2} - \tan x$ using x_1 = 3.8 as the initial guess. List each approximation until the approximation is repeated to 6 decimal places.

X1=3.8 x2=3.369190 x3=3.239422 x4=3.236756

x5=3.236755 x6=x5

2) A softball diamond has the shape of a square with sides 60 feet long. If a player is running from second base to third base at a speed of 24 ft/sec, at what rate is her distance from home plate changing when she is 20 ft from third?

Want $\frac{dL}{dt}$ at the instant y = 20. Also given $\frac{dy}{dt} = -24$.

Then $L^2 = x^2 + y^2$ then differentiate to get:

$$\frac{dL}{dt} = \frac{y\frac{dy}{dt}}{L} = \frac{(20)(-24)}{\sqrt{(20)^2 + (60)^2}} = -7.6$$

3) For $f(x) = \sin x - \cos x$ determine:

a) critical numbers

Set f’(x) = 0

f’(x) = cos x - sin x = 0

$$x = \frac{3p}{4}, \frac{7p}{4}$$

b) relative maximum

$$\sqrt{2}\ ,\ \ when\ x = \frac{3p}{4}$$

relative minimum

$$\left(\frac{7p}{4}, -\sqrt{2}\right)$$

c) intervals where f(x) is

increasing:

$$\left(0, \frac{3p}{4}\right) \cup \left(\frac{7p}{4}, 2p\right)$$

decreasing:

$$\left(\frac{3p}{4}, \frac{7p}{4}\right)$$

d) intervals where f(x) is

concave up:

$$\left(0, \frac{p}{4}\right) \cup \left(\frac{5p}{4}, 2p\right)$$

concave down:

$$\left(\frac{p}{4}, \frac{5p}{4}\right)$$

4) Determine the absolute maximum and absolute minimum values of $f(x) = x^2 - 8\ln x$ over the interval $[1, e]$.

Critical Number(s): $f'(x) = 2x - \frac{8}{x} = \frac{2x^2 - 8}{x} = 0$

So x = +/- 2, but only x = 2 is in the interval of interest.

So we need: f(1) = 1, f(e) = -0.61, and f(2) = -1.545

So the absolute max is 1 when x = 1

and the absolute min is -1.545 when x = 2.

5) Evaluate: $\lim_{x \to 0} (1 + 3x)^{1/2x}$

Rewrite $(1 + 3x) = e^{\ln(1+3x)}$ Then $(1 + 3x)^{1/2x} = e^{\left(\ln(1+3x)/2x\right)}$

so the original limit may be written as:

$\lim_{x \to 0} (1 + 3x)^{1/2x} = e^{\lim_{x \to 0}\left(\ln(1+3x)/2x\right)}$. Now consider just the exponent.

$\lim_{x \to 0} \frac{\ln(1 + 3x)}{2x} \underset{=}{LR} \lim_{x \to 0} \frac{3}{2(1 + 3x)} = \frac{3}{2}$, So $\lim_{x \to 0} (1 + 3x)^{1/2x} = e^{\left(3/2\right)}$