Math 141 Exam 1 Fall 03 Name

No work-No Credit SS#

1) Given $x^2 - x - 1 = \frac{1}{x+1}$, use the Intermediate Value Theorem to show there is a solution for 1 < x < 2.

Let $f(x) = x^2 - x - 1 - \frac{1}{x+1}$. Then f(1) = -1.5 and f(2) = $0.\overline{6}$, so there must be a "c" in the interval (1,2) such that f(c) = 0.

2) Complete this definition:

a) f(x) is continuous at a number x = a if $\lim_{x \to a} f(x) = f(a)$.

b) Let $f(x) = \begin{cases} 2, & x \le -1 \\ ax + b, & -1 < x < 3 \\ -2, & x \ge 3 \end{cases}$. Find all values of c such that f is continuous on $\Re$.

Note that for all x except –1 & 3 f(x) is continuous, so investigate there.

We need $\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (2) = 2$ = - a + b = $\lim_{x \to -1^+} (ax + b)$.

And this gives (i) 2 = -a + b, also we need

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} (ax + b) = 3a + b$ = -2 = $\lim_{x \to 3^+} (-2)$ giving (ii) 3a + b = -2.

So now we need to solve (i) and (ii) $\begin{matrix} -a + b = 2 \\ 3a + b = -2 \end{matrix}$ in the first equation, solve for b, b = a + 2 then Substitute this into the second equation, 3a + (a + 2) = -2 giving a = -1 and back substituting gives b = 1.

3) Evaluate, if possible: $\lim_{x \to 2} \frac{x^4 - 16}{x - 2}$.

$$\lim_{x \to 2} \frac{x^4 - 16}{x - 2} = \lim_{x\to 2}\frac{(x^2+4)(x-2)(x+2)}{x-2} = \lim_{x\to 2}(x^2+4)(x+2) = 32$$

4a) State the formal ε,δ definition of the limit, $\lim_{x\to a} f(x) = L$.

$\lim_{x\to a} f(x) = L$ means that for every ε > 0 there is a δ > 0 such that if
$0 < |x - a| < \delta$ then $|f(x) - L| < \varepsilon$.

b) Use part (a) to determine: $\lim_{x\to -3}\left(\frac{x-1}{2}\right) = -2$.

Proof: Let ε > 0. Choose δ = 2ε. Then whenever $0 < |x - {}^{-}3| < \delta$ we will have
$\left|\left(\frac{x-1}{2} - {}^{-}2\right)\right| < \varepsilon$.

5) Given $f(x) = \frac{2}{x}$, find f '(x) using f '(x) = $\lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$.

$$\lim_{h\to 0}\frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}\frac{\frac{2}{x+h} - \frac{2}{x}}{h} = \lim_{h\to 0}\frac{2x - 2x - 2h}{h\,x(x+h)} = \lim_{h\to 0}\frac{-2}{x(x+h)} = \frac{-2}{x^2}$$

6) Given $f(x)=\sqrt{x^3+1}$ & $f'(x)=\dfrac{3x^2}{2\sqrt{x^3+1}}$, find the linear approximation, L(x), at x = 2 . That is, find L(x), such that L(x) ≈ f(x) as long as x is near 2 .

$f(2)=\sqrt{8+1}=3$, so the point on f is (2,3) and the slope of the line tangent is f ' (2) = 2. So the equation of the line tangent is y – 3 = 2(x-2) $\Rightarrow$ y = 2x – 1.

So L(x) = 2x – 1. Then if x is very near 2, then $\sqrt{x^3+1} \approx 2x-1$.

7) Evaluate, if possible: $\lim\limits_{x\to\infty} 4000\dfrac{x^3+500x^2}{x^4+1} =$

$$\lim_{x\to\infty} 4000\frac{x^3+500x^2}{x^4+1}\left(\frac{\frac{1}{x^4}}{\frac{1}{x^4}}\right) = 4000\lim_{x\to\infty}\frac{\frac{1}{x}+\frac{500}{x^2}}{1+\frac{1}{x^4}} = 0.$$

8) For $k(x)=\dfrac{3x^2+4x+5}{x^2+8x-20}$, determine:

a) Vertical asymptotes (if any):

$k(x)=\dfrac{3x^2+4x+5}{x^2+8x-20}=\dfrac{3x^2+4x+5}{(x+10)(x-2)}$ so x = -10 & x = 2 are vertical asymptotes.

b) Horizontal asymptotes (if any):

$\lim\limits_{x\to\infty}\dfrac{3x^2+4x+5}{x^2+8x-20}\left(\dfrac{\frac{1}{x^2}}{\frac{1}{x^2}}\right)=\lim\limits_{x\to\infty}\dfrac{3+\frac{4}{x}+\frac{5}{x^2}}{1+\frac{8}{x}-\frac{20}{x^2}} = 3$. So y = 3 is a horizontal asymptote.