

Some Related Rate Problems From Old Exams

- 1) The surface area of a sphere is $S = 4\pi R^2$, where R is the radius of the sphere. If the surface area is decreasing at the rate of 6 ft^2 per second, what is the rate of change of the radius at the instance $S = 200\pi \text{ ft}^2$?

If $S = 4\pi R^2$ then $\frac{dS}{dt} = 8\pi R \frac{dR}{dt}$ and we want $\frac{dR}{dt} = \frac{dS/dt}{8\pi R}$, so now we must find R

when $S = 200\pi$. $200\pi = 4\pi R^2 \Rightarrow R = \sqrt{50}$. Then $\frac{dR}{dt} = \frac{dS/dt}{8\pi R} = \frac{-6}{8\pi\sqrt{50}} = \frac{-3}{4\pi\sqrt{50}}$.

- 2) A large red balloon is rising at the rate of 20 ft/sec . The balloon is 10 ft above the ground at the point in time that the back end of a green car is directly below the bottom of the balloon. The car is traveling at 40 ft/sec . What is the rate of change of the distance between the bottom of the balloon and the point on the ground directly below the back of the car one second after the back of the car is directly below the balloon?

Let L = desired distance, we want $\frac{dL}{dt}$ one second after the car is directly below the balloon. Given $\frac{dx}{dt} = 40$ & $\frac{dy}{dt} = 20$ where x is the direction the car travels and y is the vertical direction the balloon travels.

Then $L^2 = x^2 + y^2$, so

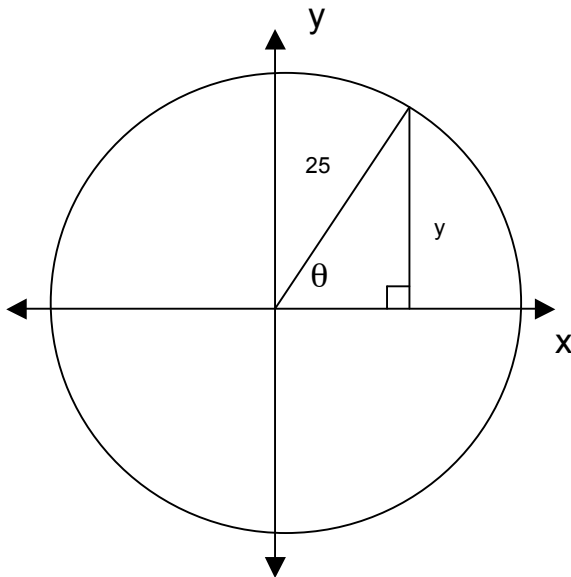
$$L \frac{dL}{dt} = x \frac{dx}{dt} + y \frac{dy}{dt} \Rightarrow \frac{dL}{dt} = \frac{x \frac{dx}{dt} + y \frac{dy}{dt}}{L} = \frac{40(40) + 30(20)}{50} = 44$$

At the instant in question we have a 30, 40, 50 right triangle.

3) A particle moves along the curve defined by $y = \frac{1}{3}x^3 - 3x$. Determine the value(s) of x at which the rate of change of its y -coordinate is equal that of its x -coordinate.

So $y = \frac{1}{3}x^3 - 3x$ then $\frac{dy}{dt} = (x^2 - 3)\frac{dx}{dt}$, now let $\frac{dy}{dt} = \frac{dx}{dt}$ giving $1 = x^2 - 3 \Rightarrow x = \pm 2$.

4) A Ferris wheel with radius 25 feet is revolving at the rate of 10 radians per minute. How fast is a passenger rising when the passenger is 15 feet higher than the center of the Ferris wheel and is rising?



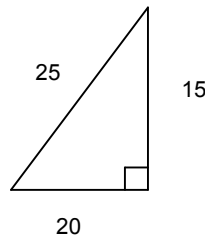
Want $\frac{dy}{dt}$ at the instant $y = 15$.

Given: $\frac{d\theta}{dt} = 10$, and radius is 25.

We have $\sin \theta = \frac{y}{25}$, so $y = 25\sin \theta$.

Then $\frac{dy}{d\theta} = 25(\cos \theta)\frac{d\theta}{dt}$

At the instant $y = 15$, we have



$\cos \theta = \frac{20}{25}$, so then $\frac{dy}{d\theta} = 25\left(\frac{20}{25}\right)(10) = 200$.