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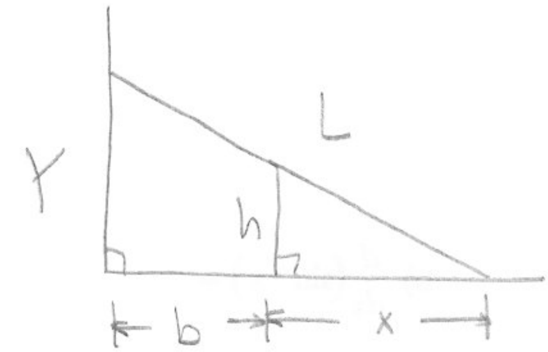

$$\frac{Y}{x+b} = \frac{h}{x}$$

$$Y = \frac{h}{x}(x+b) = h\left(1+\frac{b}{x}\right)$$

$$L^2 = Y^2 + (x+b)^2 = \left(h\left(1+\frac{b}{x}\right)\right)^2 + (x+b)^2$$

Let $f(x) = L^2 = \left(h\left(1+\frac{b}{x}\right)\right)^2 + (x+b)^2$

we will minimize $f(x)$ which minimizes L^2.

$$f'(x) = 2\left(h(1+bx^{-1})\right)\left[-bhx^{-2}\right] + 2(x+b) = 0$$

$$= h\left(1+\frac{b}{x}\right)\left[-\frac{bh}{x^2}\right] + (x+b) = 0$$

$$= h\left(\frac{x+b}{x}\right)\left[-\frac{b}{x^2}\right] + (x+b) = 0$$

$$= (x+b)\left[\frac{-bh^2}{x^3} + 1\right] = 0$$

so $\cancel{x+b=0}$ or $\frac{-bh}{x^3} + 1 = 0 \Rightarrow 1 = \frac{bh^2}{x^3}$

$\cancel{x=-b}$

so $x^3 = bh^2 \Rightarrow x = b^{1/3}h^{2/3}$

then $x+b = b^{1/3}h^{2/3} + b = b^{1/3}\left(h^{2/3} + b^{2/3}\right)$

and $Y = h\left(1+\frac{b}{b^{1/3}h^{2/3}}\right) = h + \frac{bh}{b^{1/3}h^{2/3}} = h + b^{2/3}h^{1/3}$

$$= h^{1/3}\left(h^{2/3} + b^{2/3}\right)$$

So then $(x+b)^2 = b^{2/3}\left(h^{2/3}+b^{2/3}\right)^2$

$$Y^2 = h^{2/3}\left(h^{2/3}+b^{2/3}\right)^2$$

$$f(x) = Y^2 + (x+b)^2 = h^{2/3}\underline{\left(h^{2/3}+b^{2/3}\right)^2} + b^{2/3}\underline{\left(h^{2/3}+b^{2/3}\right)^2}$$

$$f(x) = \left(h^{2/3}+b^{2/3}\right)\left(h^{2/3}+b^{2/3}\right)^2 = \left(h^{2/3}+b^{2/3}\right)^3$$

but $f(x) = L^2 \Rightarrow L = \left(h^{2/3}+b^{2/3}\right)^{3/2}$

Is this a minimum?

Yes! $f''(x) = \dfrac{bh^3(2x+b)}{x^4} + 1 > 0 \Rightarrow \underline{\cup} \Rightarrow \underline{\text{min.}}$