Chapter 4 Local Max & Mins & Inflection Points

For $f(x) = 2x^3 - 3x^2 - 12x$, determine:

a) Critical values.
b) Critical points.
c) Intervals on the x-axis where f(x) is increasing and decreasing.
d) Intervals on the x-axis where f(x) is concave up and concave down.
e) Determine point(s) of inflection, if any.
f) The relative maximum and relative minimum values of f(x).

Solutions:

a) Critical values.

The critical values are found by setting $f'(x) = 0$ and solving for x.

Here $f'(x) = 6x^2 - 6x - 12$ then we need to solve $f'(x) = 6x^2 - 6x - 12 = 0$.

$6(x^2 - x - 2) = 6(x-2)(x+1) = 0 \Rightarrow$ **x = 2 & x = -1 are the critical values.**

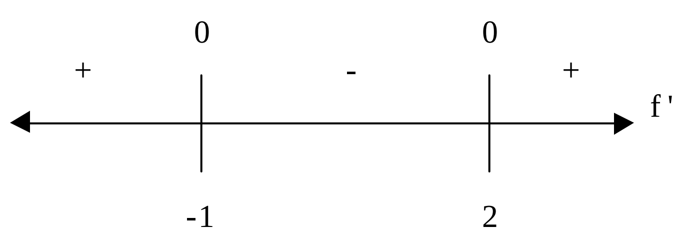
b) Critical Points.

Critical points are points on the graph of the function, f(x). So we need to substitute -1 & 2 into f(x) to get the coordinates.

(2, -20) & (-1, 7) are the critical points.

C) Intervals where f(x) is increasing and decreasing.

Recall, $f'(x) = 6x^2 - 6x - 12 = 6(x-2)(x+1)$, we will use the first derivative test here.



All we did here was select representative values from each interval and substitute into f ' (x) . For example, for the first interval on the left pick x = -2, then f '(-2) > 0. So then the first derivative is positive for any x-value to the left of -1. Since the first derivative is positive, then the function is increasing for these x-values.

f(x) is increasing for: $(-\infty,-1)\cup(2,\infty)$

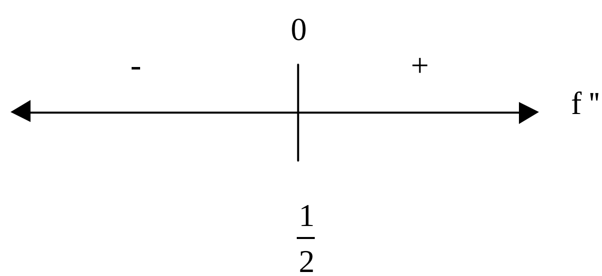
f(x) is decreasing for: $(-1,2)$**.**

D) <u>Intervals where f(x) is concave up and concave down.</u>

To discuss concavity, you need the second derivative, f '' (x). Recall f ' (x) = $6x^2 - 6x - 12$.

So, f '' (x) = 12x - 6. To get the intervals we need to set this equal to zero.

f '' (x) = 12x - 6 = 6(2x-1) = 0 $\Rightarrow$ x = $\frac{1}{2}$. Now let's mimic the first derivative test.



Here, we select any value to the left of 1/2, say x = 0 and plug into f ''(x).

f ''(0) < 0 so f ''(x) will be negative for any value we pick that is less than 1/2.

f(x) is concave up for: $\left(\frac{1}{2},\infty\right)$

f(x) is concave down for: $\left(-\infty,\frac{1}{2}\right)$**.**

E) **<u>Point(s) of inflection, if any.</u>**

For a point of inflection to exist, **<u>two</u>** things must occur:

a) There must be some x-value, c, such that f "(c) = 0
b) The second derivative must change sign at this x-value, c.

Notice here that both (a) and (b) occur. For (a), f "(1/2) = 0 and
f "(x) changes from negative to positive as **x** passes through $\frac{1}{2}$.

Now be careful, we need the ***point*** of inflection that is we need a coordinate on f(x) where the function changes concavity.

Here $\left(\frac{1}{2}, -6.5\right)$ **is the *point* of inflection.**

F) **<u>The relative maximum and relative minimum values of f(x).</u>**

To determine the relative maximum and relative minimum values of f(x) we need the Second Derivative Test.

Second Derivative Test:

a) if f "(c) < 0, then the critical point (c, f (c)) is a relative maximum.
b) if f "(c) > 0, then the critical point (c, f (c)) is a relative minimum.
c) if f "(c) = 0, then the test fails.

f ''(-1) < 0, so f(-1) = 7 is a relative maximum value of f(x). Or, (-1, 7) is a relative maximum point on the graph of f(x).

f ''(2) > 0, so f(2) = -20 is a relative minimum value of f(x). Or, (2, -20) is a relative minimum point on the graph of f(x).