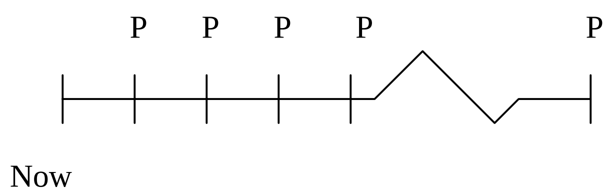
Section 2.4 Present Value of an Annuity Math 123 F05

Consider this situation: You have found the house of your dreams. You will need to finance \$225,000 at 4.5% compounded monthly for 30 years. What is your payment each month, with the first payment due one month from today?

The time line looks like this:



You owe \$225,000 now and need to determine the value of P. When the last payment, P, is made the debt is completely amortized. Note that the payment, P, includes interest plus an amount that will reduce the debt.

When derived the formula needed is: $V = \dfrac{P\left(1-\left(1+\frac{r}{m}\right)^{-mt}\right)}{\left(\frac{r}{m}\right)}$.

Here V is referred to as the *Present Value of an Annuity*, P is the periodic payment with r, m and t having the usual meanings.

After entering the formula into the calculator we need to solve for P:

V = 225000 We find P = 1140.0419471083 and **we always round up!!!!!**
R = 0.045
M = 12
T = 30 so P = 1140.05.

So the payment will be \$1140.05 for the next 30 years (~ 360* payments).

*Since we rounded up the last payment will be a little less than \$1140.05.

Question: Over the life of the note what is the total(approximately) paid?

We have 360 payments of $1140.05, so we have 360 * $1140.05 = $410,418.

Question: What is the interest paid? $410,418 - $225,000 = $ 185,418.

What is going on here? These ideas are the key for this section.

a) You borrowed $225,000, now the lender did not have to loan you that money! Instead the lender could have invested the money at 4.5% compounded monthly for 30 years. Using the compound interest formula:

P = 225000 we find A = $865,732.06.
R = 0.045
M = 12
T = 30

b) Assuming you were loaned the money, these payments of $1140.05 (from the point of view of the lender) could then be invested forming a sinking fund (S = from section 2.3). The total accumulation of these 360 deposits will be:

P = 1140.05 this gives S = $865,738.18.
R = 0.045
M = 12
T = 30

The small difference in the values is just the result of when we rounded-up to get $1140.05.

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That is to say the values in (a) and (b) are the same! If this were not so, then there would be no reason for the lender to loan you the $225,000.

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Question: What is the effect of the term, T, on the amount of your payment and total interest paid?

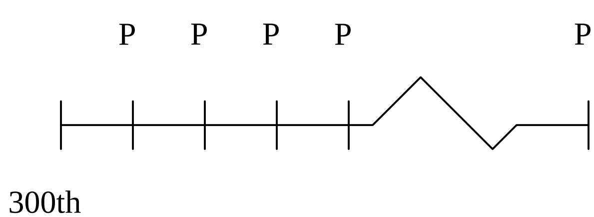
In the table below, let T vary in the formula, $V = \dfrac{P\left(1-\left(1+\dfrac{r}{m}\right)^{-mt}\right)}{\left(\dfrac{r}{m}\right)}$, hold the other values constant.

T	Payment (P)	Interest
30	$1140.05	$185418.00
20	$1423.47	$116632.80
15	$1721.24	$ 84823.20

Notice that financing for 15 years instead of 30 years will cost you about another $600 a month but will save you over $100,000 in interest!!!!!!!!!!!

Question: Suppose you finance $225,000 for 30 years at 4.5% compounded monthly. You make payments faithfully for 25 years. Approximately how much do you still owe on the loan?

The situation is illustrated by the timeline below.



Here we have 60 payments remaining. This is exactly the form of a Present Value of an Annuity problem. Here we need to find V.

P = 1140.04 This gives V = $61151.58.
R = 0.045
M = 12
T = 5

So after making payments for 25 years we still owe over $60,000!

Question: Suppose your budget allows you to pay an extra $100 on this note. When will the note now be paid off?

V = 225000
P = 1140.05 + 100
R = 0.045
M = 12
T = 25.397178331043

We see that we pay off the 30-year note almost 5 years early and save almost 1140.05 * 60 = $68403.

Amortization Schedule

Let's look at the first three payments in the amortization schedule for a 30 year note of $225,000 at 4.5% compounded monthly, where the payment is $1140.05.

Payment Number	Payment $	Interest	$ To Reduce Debt(Principal)	Unpaid Balance
0				225000.00
1	1140.05	843.75	296.30	224703.70
2	1140.05	842.64	297.41	224406.29
3	1140.05	841.52	298.53	224107.76

How did we get these numbers? Row 1 is just a statement that we owe $225,000.

Look at row 2. Multiply $225,000 * $\left(\frac{0.045}{12}\right)$ = 843.75, this is the interest due for the first month. The $\left(\frac{0.045}{12}\right)$ is the interest rate per compounding period (interest rate per month). Notice the payment of $1140.05 is split into two parts. The remainder of the payment will go to reduce the debt. Finally, subtract $296.30 from $225,000 to get the unpaid balance (amount you now owe) after one payment.

In the above table you will round using the normal rounding rule.

Included here is a spreadsheet with the entire amortization schedule for a $225,000 loan at 4.5% compounded monthly for 30 years.

Complete Amortization Schedule.