

Homework #8

6.34. As

$$f(x|n, \theta) = \binom{n}{x} \theta^x (1 - \theta)^{n-x}$$

and

$$f(x, n|\theta) = p_n \binom{n}{x} \theta^x (1 - \theta)^{n-x},$$

we have

$$\frac{f(x, n|\theta)}{f(x|n, \theta)} = p_n.$$

6.35. Sample points: $(1, F)$, $(2, SF)$, $(3, SSS)$, $(3, SSF)$.

$$f(x_1|p) = 1 - p, \quad f(x_2|p) = p(1 - p), \quad f(x_3|p) = p^3 \text{ and } f(x_4|p) = p^2(1 - p).$$

Also

$$f(F|1, p) = 1 - p, \quad f(SF|2, p) = p(1 - p), \quad f(SSS|3, p) = p^3 \text{ and } f(SSF|3, p) = p^2(1 - p).$$

The likelihood function does not depend on the fact that the sample size was determined by the data.

6.39. a) Note that

$$y = \frac{9}{5}x + 32.$$

Thus

$$\begin{aligned} \frac{5}{9}(T^*(y) - 32) &= \frac{5}{9}(.5(y + 212) - 32) \\ &= \frac{5}{9}(.5y + 74) \\ &= \frac{5}{18} \left(\frac{9}{5}x + 90 \right) \\ &= .5x + .5(100) = T(x). \end{aligned}$$

b)

$$T^*(x) = .5(x + 212) \neq .5(x + 100).$$

6.41. a)

$$T^*(g_a(x)) = \bar{g}_a(T(x)) = T(x) + a.$$

So

$$T(x) = T^*(y) - a.$$

b) Since X and Y have the same structure, FIP implies that $T(x) = T^*(x)$.

c)

$$W(g_a(x)) = \bar{x} + a = W(x) + a = \bar{g}_a(W(x)).$$

6.42. a) As

$$Y = g_{a,c}(X) \sim \frac{1}{c\sigma} f\left(\frac{y - (a + c\theta)}{c\sigma}\right),$$

we have

$$\bar{g}_{a,c}(\theta) = a + c\theta \text{ and } \bar{g}_a(\theta) = a + \theta.$$

Note that

$$W(g_{a,c}(x)) = c\bar{x} + a + k,$$

we see that

$$\begin{aligned} \bar{g}_{a,c}(W(x)) &= \bar{g}_{a,c}(\bar{x} + k) \\ &= a + c(\bar{x} + k) = c\bar{x} + a + ck \\ &\neq W(g_{a,c}(x)). \end{aligned}$$

It is clear that

$$\bar{g}_a(W(x)) = W(g_a(x)).$$

b) Note that

$$W(a + cx) = W(g_{a,c}(x)) = \bar{g}_{a,c}(W(x)) = cW(x) + a.$$

Let $\tilde{x}_i = x_i - \bar{x}$. Then

$$W(x) = W(\tilde{x}) + \bar{x},$$

and hence,

$$E_\theta W(X) = E_\theta W(\tilde{X}) + E_\theta \bar{X}.$$

It is unbiased iff

$$E_\theta W(\tilde{X}) = 0 \text{ and } E_\theta \bar{X} = \theta.$$

6.43. Similar to 6.42.