Homework #1

5.1. Suppose the size is n. Let X be the number of color-blind person taken. Then $X \sim B(n, .01)$.

$$\begin{aligned} P(X \geq 1) &= 1 - P(X = 0) \\ &= 1 - .99^n > .95. \end{aligned}$$

So

$$n > \frac{\log .05}{\log .99} = 298.07.$$

Thus $n = 299$.

5.2. a) For $k = 2, 3, \cdots$,

$$\begin{aligned} P(N = k) &= P(X_2 \leq X_1, \cdots, X_{k-1} \leq X_1,\ X_k > X_1) \\ &= \int_0^\infty F(x)^{k-2}(1 - F(x))f(x)dx \\ &\overset{u=F(x)}{=} \int_0^1 u^{k-2}(1-u)du \\ &= \frac{1}{k-1} - \frac{1}{k} = \frac{1}{k(k-1)}. \end{aligned}$$

b)

$$E(N) = \sum_{k=2}^{\infty} \frac{1}{k-1} = \infty.$$

5.3.

$$\sum_{i=1}^{n} Y_i \sim B(n, p)$$

where

$$p = P(X_i > \mu) = 1 - F_X(\mu).$$

5.4. a)

$$\begin{aligned} P(X_1 = x_1, \cdots, X_k = x_k) &= \int_0^1 p^{x_1}(1-p)^{1-x_1} \cdots p^{x_k}(1-p)^{1-x_k} dp \\ &= \int_0^1 p^t (1-p)^{k-p} dp \\ &= \beta(t+1, k-t+1) = \frac{\gamma(t+1)\Gamma(k-t+1)}{\Gamma(k+2)} \\ &= \frac{t!(k-t)!}{(k+1)!}. \end{aligned}$$

b) As

$$P(X_i = x_i) = \frac{1}{2},$$

we see that

$$\Pi_{i=1}^n P(X_i = x_i) = \frac{1}{2^n}.$$

Thus

$$P(X_1 = x_1, \cdots, X_k = x_n) \neq \Pi_{i=1}^n P(X_i = x_i).$$