MATH 323 First Midterm Exam

February 9, 2009

Name _____

ID number _____

1. You order two pizzas from two pizza parlors, Morticia's and Gomez's. The probability that Morticia's delivers in 30 minites or less is .95 and the probability that Gomez's delivers in under 30 minutes is .90. Assumming the two pizza companies deliver independently, find the following proabibility:

   (a) (6 points) Both pizzas arrive in 30 minutes or less.

   Solution:
   $$P(MG) = P(M)P(G) = .95 \times .9 = .855$$

   (b) (6 points) Neither pizza is delivered in under 30 minutes.

   Solution:
   $$P(\bar{M}\bar{G}) = (1 - .95)(1 - .9) = .005$$

   (c) (6 points) At least one of the pizza is delivered on time.

   Solution:
   $$\begin{aligned} P(M \cup G) &= P(M) + P(G) - P(MG) \\ &= .95 + .9 - .855 \\ &= .995 \end{aligned}$$

2. Jerry gives his class a set of 10 problems with the information that the final exam will consist of a random selection of 5 of them. A student has figured out how to do 7 of the 10 problems.

   (a) (12 points) What is the probability that he or she will answer correctly all the 5 problems?

   Solution:

$$p(5) = \frac{\binom{7}{5}}{\binom{10}{5}} = \frac{1}{12}$$

   (b) (8 points) What is the probability that he or she will answer correctly four or more problems?

   Solution:

$$p(4) = \frac{\binom{7}{4}\binom{3}{1}}{\binom{10}{5}} = \frac{5}{12}$$

   So

$$P(\text{at least } 4) = p(4) + p(5) = \frac{1}{12} + \frac{5}{12} = \frac{1}{2}$$

3. Jerry Commutes to work on two different routes 1 and 2. If Jerry comes home by route 1, then he will be home before 6 p.m. with probability .9. If Jerry chooses route 2, then he will be home before 6 p.m. with probability .7. Jerry chooses route 1 with probability .6. Define the following two events.
   A=[Jerry is home before 6 p.m.]
   B=[Jerry chooses route 1]

   (a) (6 points) Determine $P(AB)$, the probability that Jerry chooses route 1 and is home from his office before 6 p.m.

   Solution:
   $$P(AB) = P(B)P(A|B) = .6 \times .9 = .54$$

   (b) (8 points) Determine P(A), the probability that Jerry is home from his office before 6.p.m. (Hint: calculate $P(A\bar{B})$ first.)

   Solution:
   $$P(A\bar{B}) = P(\bar{B})P(A|\bar{B}) = .4 \times .7 = .28$$

   so
   $$P(A) = P(AB) + P(A\bar{B}) = .54 + .28 = .82$$

   (c) (8 points) If Jerry is home before 6 p.m. today, what is the probability that he took route 2?

   Solution:
   $$P(\bar{B}|A) = \frac{P(A\bar{B})}{P(A)} = \frac{.28}{.82} = .3415$$

4. Jim and Ann play the following game. Jim rolls a fair die (six-faced, which can be 1,2,3,4,5 or 6 with equal probability 1/6) three times and records the number of times the die shows an **even** number of dots (that is, 2, 4 or 6) as X. The three outcomes of his die are independent. Independently, Ann rolls a die three times and records the number of times the die shows an **odd** number of dots (that is, 1,3 or 5) as Y. The three outcomes of her die are independent. Let $W = X + Y$.

   (a) (6 points) What is the distribution of X? and of Y?

   Solution: X is binomial with $n = 3$ and $p = \frac{1}{2}$ (success means even), Y is binomial with $n = 3$ and $p = \frac{1}{2}$ (success means odd).

   (b) (6 points) What is the distribution of W?

   Solution: W is binomial with $n = 6$ and $p = \frac{1}{2}$.

   (c) (6 points) Calculate $P(W = 4)$.

   Solution:

$$P(W = 4) = \binom{6}{4} \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^2 = \frac{15}{64}$$

   (d) (6 points) Determine $E(W)$, the expected value of W, the variance of W, $Var(W)$, and $SD(W)$, the standard deviation of W.

   Solution:

$$E(W) = 6 \times \frac{1}{2} = 3$$

$$Var(W) = 6 \times \frac{1}{2} \times \frac{1}{2} = 1.5$$

$$SD(W) = \sqrt{1.5} = 1.2247$$

5. Suppose that in a city the number of suicides can be approximated by a Poisson process with $\lambda = .33$ per month (assuming 4 weeks in a month).

   (a) (8 points) What is the probability of two suicides in the next week?

   Solution: Let X be the number of suicides in next week. Then, X is Poisson with $\lambda_1 = \frac{1}{4}\lambda = .0825$

   $$p(2) = \frac{.0825^2}{2!}e^{-.0825} = .0031$$

   (b) (8 points) What is the probability that two suicides would occur in the next week **and** one would occur in the week after the next?

   Solution: Let Y be the number of suicides in the week after next one.

   $$p(1) = \frac{.0825^1}{1!}e^{-.0825} = .076$$

   So

   $$p(X = 2, Y = 1) = p(2)p(1) = .0031 \times .076 = .00024$$