Review for Chapter 8

1. (a) Null hypothesis H_0: believe until disproved
   (b) Alternative hypothesis H_a: to be proved
   (c) Type I error: reject H_0 when H_0 is true
   (d) Type II error: fail to reject H_0 when H_0 is false
   (e) Significant level α: pre-specified so that Type I error $\lambda\alpha$
   (f) P-value: smallest α for you to be able to reject H_0

2. Test mean, variance and p

Case	Null H_0	Test Stat	Rejection Region	$d.f.$
large n	$\mu = 10$		$\|Z\| > z_{\alpha/2}$	
general	$\mu \leq 10$	$Z = \frac{\bar{x}-10}{\sigma/\sqrt{n}}$	$Z > z_\alpha$	
distribution	$\mu \geq 10$		$Z < -z_\alpha$	
binomial	$p = .2$		$\|Z\| > z_{\alpha/2}$	
large n	$p \leq .2$	$Z = \frac{\hat{p}-.2}{\sqrt{\frac{.2(1-.2)}{n}}}$	$Z > z_\alpha$	
	$p \geq .2$		$Z < -z_\alpha$	
	$\mu = 10$		$\|T\| > t_{\alpha/2}$	
normal	$\mu = 10$	$T = \frac{\bar{x}-10}{s/\sqrt{n}}$	$T > t_\alpha$	$\nu = n - 1$
	$\mu \geq 10$		$T < -t_\alpha$	
normal	$\sigma^2 = 10$		$U < \chi^2_{1-\alpha/2}$ or $U > \chi^2_{\alpha/2}$	
	$\sigma^2 \leq 10$	$U = \frac{(n-1)s^2}{10}$	$U > \chi^2_\alpha$	$\nu = n - 1$
	$\sigma^2 \geq 10$		$U < \chi^2_{1-\alpha}$	

3. χ^2-tests

   (a) $(X_1, \cdots, X_k)$ multinomial

$$H_0: \quad p_1 = p_1^0, \ \cdots, \ p_k = p_k^0$$

$$X^2 = \sum_{i=1}^{k} \frac{(X_i - np_i^0)^2}{np_i^0}$$

   Reject H_0 if $X^2 > \chi^2_\alpha$, $\nu = k - 1$

(b) $X_1, \cdots, X_k$ independent binomials

$$H_0: \quad p_1 = \cdots = p_k$$

$$X^2 = \sum_{i=1}^{k} \frac{(X_i - n_i\hat{p})^2}{n_i\hat{p}(1-\hat{p})}$$

where

$$\hat{p} = \sum_{i=1}^{k} X_i \Big/ \sum_{i=1}^{k} n_i$$

Reject H_0 if $X^2 > \chi^2_\alpha$, $\nu = k - 1$

(c) Contingency table

$$H_0: \quad \text{independent}$$

$$X^2 = \sum_{i=1}^{r}\sum_{j=1}^{c} \frac{(X_{ij} - n\hat{p}_{i\cdot}\hat{p}_{\cdot j})^2}{n\hat{p}_{i\cdot}\hat{p}_{\cdot j}}$$

Reject H_0 if $X^2 > \chi^2_\alpha$, $\nu = (r-1)(c-1)$

(d) Y take $y_1, \; y_2, \; \cdots$

$$H_0: \quad P(Y = y_i) = p_i, \quad i = 1, 2, \cdots, k.$$

$$F_i = \text{number of times } y_i \text{ observed}$$

$$X^2 = \sum_{i=1}^{k} \frac{(F_i - n\hat{p}_i)^2}{n\hat{p}_i}$$

Reject H_0 if $X^2 > \chi^2_\alpha$, $\nu = k - 1 - \text{number of parameters estimated}$