Review for the second midterm

1. X is a continuous r.v. with p.d.f. f if
   - $f(x) \geq 0$ for all x
   - $\int_{-\infty}^{\infty} f(x)dx = 1$
   - $P(a \leq X \leq b) = \int_a^b f(x)dx$
2. The d.f. of X is
$$F(b) = \int_{-\infty}^{b} f(x)dx$$
   - $F'(x) = f(x)$
3. Expectation
$$\mu = \mathbb{E}(X) = \int_{-\infty}^{\infty} xf(x)dx$$
   - $\mathbb{E}(g(X)) = \int_{-\infty}^{\infty} g(x)f(x)dx$
   - $V(X) = \mathbb{E}((X-\mu)^2) = \mathbb{E}(X^2) - \mu^2$
   - If $a,\ b$ are constants, then
$$\mathbb{E}(aX+b) = a\mathbb{E}(X) + b$$
$$V(aX+b) = a^2V(X)$$
4. X has uniform distribution on (a,b) if
$$f(x) = \frac{1}{b-a}, \qquad a < x < b$$
   - $\mu = \frac{a+b}{2}$, $\sigma^2 = \frac{(b-a)^2}{12}$
5. X has exponential distribution with parameter θ
$$f(x) = \frac{1}{\theta}e^{-\frac{x}{\theta}}, \qquad x > 0$$
   - $\mu = \theta$, $\sigma^2 = \theta^2$

- $$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$$
$$\Gamma(\alpha+1) = \alpha\Gamma(\alpha), \qquad \Gamma(n) = (n-1)!$$
$$\int_0^\infty x^{\alpha-1} e^{-\frac{x}{\beta}} dx = \beta^\alpha \Gamma(\alpha)$$
- Waiting time between two Poisson points is of exponential distribution with $\theta = \frac{1}{\lambda}$

6. X has gamma distribution with parameters α, β if
$$f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}}, \qquad x > 0$$
   - $\mu = \alpha\beta$, $\sigma^2 = \alpha\beta^2$

7. X has normal distribution with parameters μ, σ^2 if
$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$
   - If $\mu = 0$ and $\sigma = 1$, then it is called the standard normal
   - If $a,\ b$ are constants, then $aX + b$ is also normal
   - $Z = \frac{X-\mu}{\sigma}$ is standard normal
   - Use table

8. Joint p.f. or p.d.f. $p(x_1, x_2),\ f(x_1, x_2)$
   - Marginal
$$p_1(x_1) = \sum_{x_2} p(x_1, x_2)$$
$$f_1(x_1) = \int_{-\infty}^\infty f(x_1, x_2) dx_2$$
   - $$P(a_1 < X_1 < a_2,\ b_1 < X_2 < b_2) = \int_{b_1}^{b_2} \int_{a_1}^{a_2} f(x_1, x_2) dx_1 dx_2$$

- Conditional

$$p(x_1|x_2) = \frac{p(x_1, x_2)}{p_2(x_2)}$$

- $X_1,\ X_2$ indep. if and only if $f(x_1, x_2) = f_1(x_1)f_2(x_2)$

9.

$$\mathbb{E}(g(X_1, X_2)) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x_1, x_2)f(x_1, x_2)dx_1dx_2$$

- If $X_1,\ X_2$ indep., then

$$\mathbb{E}(h(X_1)\ell(X_2)) = \mathbb{E}(h(X_1))\mathbb{E}(\ell(X_2))$$

10.

$$\begin{aligned} Cov(X_1, X_2) &= \mathbb{E}((X_1 - \mu_1)(X_2 - \mu_2)) \\ &= \mathbb{E}(X_1X_2) - \mu_1\mu_2 \end{aligned}$$

- If $X_1,\ X_2$ indep., then $Cov(X_1, X_2) = 0$
- $\rho = \frac{Cov(X_1,X_2)}{\sqrt{V(X_1)V(X_2)}}$ is the correlation coefficient

11.

$$Cov\left(\sum_{i=1}^{n} a_iY_i, \sum_{j=1}^{m} b_jX_j\right) = \sum_{i=1}^{n}\sum_{j=1}^{m} a_ib_jCov(Y_i, X_j)$$

$$V\left(\sum_{j=1}^{m} b_jX_j\right) = \sum_{j=1}^{m} b_j^2V(X_j) + 2\sum_{i<j} b_ib_jCov(X_i, X_j)$$

12. Multinomial distribution

- n trials, each results in k possible outcomes with probabilities $p_1, \cdots, p_k$, Y_i = occurrence of ith outcome
- $\mathbb{E}(Y_i) = np_i$, $V(Y_i) = np_i(1 - p_i)$
- $Cov(Y_i, Y_j) = -2np_ip_j$, $i \neq j$

13.

$$\mathbb{E}(X_1|X_2 = x_2) = \int_{-\infty}^{\infty} x_1f(x_1|x_2)dx_1$$