Review for the first midterm

1. Venn diagram
   - $A \cup B$, $AB$, $\bar{A}$
   - $A(B \cup C) = (AB) \cup (AC)$
     $A \cup (BC) = (A \cup B)(A \cup C)$
   - $\overline{A \cup B} = \bar{A}\bar{B}$
     $\overline{AB} = \bar{A} \cup \bar{B}$
2. Sample space, events
3. Probability
   (a) $P(A) \geq 0$ for any event $A$
   (b) $P(S) = 1$
   (c) If $A_1,\ A_2,\ \cdots$ are mutually exclusive, then
   $$P\left(\cup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i).$$
4. Counting rules
   (a) If the first task has $n_1$ outcomes, the second task has $n_2$ outcomes, then the experiment of two tasks has $n_1 n_2$ outcomes
   (b) The # of permutations of $r$ objects selected from $n$ distinct objects is
   $$P_r^n = n(n-1)\cdots(n-r+1) = \frac{n!}{(n-r)!}$$
   (c) The # of combinations of $r$ objects selected from $n$ distinct objects is
   $$\binom{n}{r} = \frac{n(n-1)\cdots(n-r+1)}{r!} = \frac{n!}{r!(n-r)!}$$
   (d) The # of ways of partitioning $n$ objects into $k$ groups containing $n_1,\ \cdots,\ n_k$ objects respectively is
   $$\frac{n!}{n_1! \cdots n_k!}$$

5. Conditional probability $P(A|B) = \frac{P(AB)}{P(B)}$
6. Rules of probability
   (a) $P(\bar{A}) = 1 - P(A)$
   (b)
   $$P(A \cup B) = P(A) + P(B) - P(AB)$$
   $$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(AB) - P(BC) - P(CA) + P(ABC)$$
   (c)
   $$P(AB) = P(A)P(B|A)$$
   (d) Bayes' rule: If $B_1, \cdots, B_k$ is a partition of $S$, then
   $$P(B_1|A) = \frac{P(B_1)P(A|B_1)}{P(B_1)P(A|B_1) + \cdots + P(B_k)P(A|B_k)}$$
7. A random variable is a function on $S$. There are two types: discrete and continuous
   - p.f. of $X$ is $p(x) = P(X = x)$. Properties
     - $p(x) \geq 0$
     - $\sum_x p(x) = 1$.
   - cdf of $X$ is $F(x) = P(X \leq x)$. Note
     $$F(x) = \sum_{a \leq x} p(a)$$
   - mean
     $$\mu = \sum_x xp(x)$$
     variance
     $$\sigma^2 = \sum_x (x - \mu)^2 p(x)$$
8. Properties
   -
     $$\mathbb{E}(g(X)) = \sum_x g(x)p(x)$$

- $$\mathbb{E}(aX+b) = a\mathbb{E}(X) + b$$
  $$Var(aX+b) = a^2 var(X)$$

- $$\sigma^2 = \mathbb{E}(X^2) - \mu^2$$

- $$P(|X-\mu| < k\sigma) \geq 1 - \frac{1}{k^2}$$

9. Distributions

   (a) Bernoulli
   $$p(x) = p^x(1-p)^{1-x}, \qquad x = 0, 1.$$
   $$\mu = p, \qquad \sigma^2 = p(1-p).$$

   (b) Binomial: # of successes in $n$ independent trials, each has two outcomes "success and failure"
   $$p(y) = \binom{n}{y} p^y (1-p)^{n-y}, \qquad y = 0, 1, \cdots, n.$$
   $$\mu = np, \qquad \sigma^2 = np(1-p).$$

   (c) Geometric: # of trials until one success
   $$p(x) = (1-p)^{x-1}p, \qquad x = 1, 2, \cdots$$
   $$\mu = \frac{1}{p}, \qquad \sigma^2 = \frac{1-p}{p^2}$$

   (d) Poisson: # of objects in a region divisible continuously.
   $$p(x) = \frac{\lambda^x}{x!}e^{-\lambda}, \qquad x = 0, 1, 2, \cdots$$
   $$\mu = \sigma^2 = \lambda$$

   (e) hypergeometric
   $$p(x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}, \qquad x = 0, 1, \cdots, k$$
   $$\mu = n\frac{k}{N}, \qquad \sigma^2 = n \cdot \frac{k}{N}\left(1 - \frac{k}{N}\right)\frac{N-n}{N-1}$$