Math 323 Homework # 8

4.46. $X \sim \text{gamma}(600, 50)$
a)

$$\mathbb{E}(X) = \alpha\beta = 600 \times 50 = 30000$$
$$V(X) = \alpha\beta^2 = 600 \times 50^2 = 1500000$$

b)

$$\begin{aligned} P(X > 35000) &= P(X - 30000 > 5000) \\ &\leq P\left(|X - 30000| > k\sqrt{1500000}\right) \leq \frac{1}{k^2} \end{aligned}$$

where

$$k = \frac{5000}{\sqrt{1500000}}.$$

Thus,

$$P(X > 35000) \leq \frac{1}{k^2} = \frac{1500000}{5000^2} = .06$$

Not many.
4.48. $X_1 =$ first waiting time $\sim$ *expo.* $\theta = \frac{1}{2}$. Thus, $X_1 \sim gamma\left(1, \frac{1}{2}\right)$
a) $X = X_1 + X_2 \sim gamma\left(2, \frac{1}{2}\right)$. Thus

$$\mathbb{E}(X) = \alpha\beta = 1$$

and

$$V(X) = \alpha\beta^2 = 2 \times \left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

The p.d.f. is

$$f(x) = 4xe^{-2x}, \qquad x > 0.$$

b) $Y = X_1 + X_2 + X_3 \sim gamma\left(3, \frac{1}{2}\right)$. Thus

$$\mathbb{E}(Y) = \alpha\beta = \frac{3}{2}$$

and

$$V(Y) = \alpha\beta^2 = 3 \times \left(\frac{1}{2}\right)^2 = \frac{3}{4}$$

The p.d.f. is

$$f(y) = 4y^2e^{-2y}, \qquad y > 0.$$

4.55. c)

$$P(.3 \leq Z \leq 1.56) = .4406 - .1179 = .3227$$

e)

$$P(-2 \leq Z \leq -1.56) = P(1.56 < Z < 2) = .4772 - .4406 = .0366$$

4.56. f)

$$P(0 < Z < z_0) = \frac{.95}{2} = .475$$
$$z_0 = 1.96$$

4.57. $X \sim N(400, 20^2)$

$$\begin{aligned} P(X > 450) &= P\left(Z > \frac{450 - 400}{20}\right) = P(Z > 2.5) \\ &= .5 - .4938 = .0062 \end{aligned}$$

4.61. $X \sim N(.13, .005^2)$

a)

$$\begin{aligned} P(.13 < X < .14) &= P\left(\frac{.12 - .13}{.005} < Z < \frac{.14 - .13}{.005}\right) \\ &= P(-2 < Z < 2) \\ &= 2 \times .4772 = .9544 \end{aligned}$$

b)

$$P(\text{all pass}) = .9544^4 = .8297$$

4.66. $X \sim N(1500, 200^2)$

a)

$$\begin{aligned} P(X < 1000) &= P\left(Z < \frac{1000 - 1500}{200}\right) = P(Z < -2.5) \\ &= .5 - .4938 = .0062 \end{aligned}$$

b)

$$P(X < x_0) = .05$$
$$P\left(Z < \frac{x_0 - 1500}{200}\right) = .05$$
$$\frac{x_0 - 1500}{200} = -1.645$$
$$x_0 = 1500 - 200 \times 1.645 = 1171$$