

Math 323 Homework # 7

4.9.

$$\mathbb{E}(X) = \frac{59 + 61}{2} = 60.$$

$$\text{Var}(X) = \frac{(61 - 59)^2}{12} = \frac{1}{3}$$

4.12.

$$\begin{aligned} \mathbb{E}(X) &= \int_0^4 x \frac{3}{64} x^2 (4 - x) dx = \frac{3}{64} \int_0^4 (4x^3 - x^4) dx \\ &= \frac{3}{64} \left(x^4 - \frac{1}{5} x^5 \right) \Big|_0^4 = \frac{3}{64} \left(4^4 - \frac{1}{5} \times 4^5 \right) \\ &= \frac{12}{5} \end{aligned}$$

$$\begin{aligned} \mathbb{E}(X^2) &= \int_0^4 x^2 \frac{3}{64} x^2 (4 - x) dx = \frac{3}{64} \int_0^4 (4x^4 - x^5) dx \\ &= \frac{3}{64} \left(\frac{4}{5} x^5 - \frac{1}{6} x^6 \right) \Big|_0^4 = \frac{3}{64} \left(\frac{4}{5} \times 4^5 - \frac{1}{6} \times 4^6 \right) \\ &= \frac{32}{5} \end{aligned}$$

$$\text{Var}(X) = \frac{32}{5} - \left(\frac{12}{5} \right)^2 = \frac{16}{25}$$

b) As $C = 200X$,

$$\mathbb{E}(C) = 200 \times \frac{12}{5} = \$480$$

and

$$\text{Var}(C) = 200^2 \times \frac{16}{25} = 25600$$

c)

$$\begin{aligned} P(C > 600) &= P(X > 3) = \int_3^4 \frac{3}{64} x^2 (4 - x) dx \\ &= \frac{3}{64} \int_3^4 (4x^2 - x^3) dx = \frac{3}{64} \left(\frac{4}{3} x^3 - \frac{1}{4} x^4 \right) \Big|_3^4 = .26 \end{aligned}$$

Not often

4.17. $X \sim U(0, 500)$

a)

$$P(X > 500 - 25) = \frac{25}{500} = \frac{1}{20}$$

b)

$$P(X < 25) = \frac{1}{20}$$

c)

$$P(X < 250) = \frac{250}{500} = \frac{1}{2}$$

4.21. Let X be the time of the defective item. Then, X is uniform over $(0, 8)$.

Thus a)

$$P(X < 1) = \frac{1 - 0}{8} = \frac{1}{8}$$

b)

$$P(7 < X < 8) = \frac{8 - 7}{8} = \frac{1}{8}$$

c)

$$P(X < 5 | X > 4) = \frac{P(4 < X < 5)}{P(X > 4)} = \frac{\frac{1}{8}}{\frac{1}{2}} = \frac{1}{4}$$

4.31. $X \sim \text{expo.}(2.4)$ a)

$$P(X > 3) = e^{-\frac{3}{2.4}} = .2865$$

b)

$$P(2 < X < 3) = P(X > 2) - P(X > 3) = e^{-\frac{2}{2.4}} - e^{-\frac{3}{2.4}} = .1481$$

4.35. a)

$$\mathbb{E}(X) = 10$$

$$\begin{aligned}\mathbb{E}(X^2) &= \int_0^\infty x^2 \frac{1}{10} e^{-\frac{x}{10}} dx = 10^2 \int_0^\infty y^2 e^{-y} dy \\ &= 10^2 \gamma(3) = 200\end{aligned}$$

$$\mathbb{E}(C) = 100 + 10 \times 10 + 3 = 1100$$

$$\begin{aligned}\mathbb{E}(X^3) &= \int_0^\infty x^3 \frac{1}{10} e^{-\frac{x}{10}} dx = 10^3 \int_0^\infty y^3 e^{-y} dy \\ &= 10^3 \gamma(4) = 6000\end{aligned}$$

Similarly

$$\mathbb{E}(X^4) = 10^4 \gamma(5) = 240000$$

Then

$$\begin{aligned}\mathbb{E}((C - 10)^2) &= \mathbb{E}(1600X^2 + 240X^3 + 9X^4) \\ &= 1600 \times 200 + 240 \times 60000 + 9 \times 240000 \\ &= 3920000\end{aligned}$$

Thus

$$\begin{aligned}Var(C) &= Var(C - 100) = \mathbb{E}((C - 10)^2) - (\mathbb{E}(C - 100))^2 \\ &= 3920000 - 1000^2 = 2920000\end{aligned}$$

b)

$$\begin{aligned}P(C > 2000) &= P(3X^2 + 40X - 1900 > 0) \\ &= P\left(X > \frac{-40 + \sqrt{1600 + 12 \times 1900}}{6}\right) \\ &= P(X > 19.367) = e^{-1.9367} = .1442\end{aligned}$$