

Math 323 Homework # 6

P190. 4.2. a)

$$\begin{aligned} 1 &= \int_0^1 kx(1-x)dx = k \int_0^1 (x-x^2)dx \\ &= k \left(\frac{1}{2}x^2 - \frac{1}{3}x^3 \right) \Big|_0^1 = k \left(\frac{1}{2} - \frac{1}{3} \right) = \frac{k}{6}. \end{aligned}$$

Hence

$$k = 6.$$

d) For $0 < x < 1$,

$$\begin{aligned} F(x) &= \int_0^x 6u(1-u)du = (3u^2 - 2u^3) \Big|_0^x \\ &= 3x^2 - 2x^3 = x^2(3 - 2x). \end{aligned}$$

b)

$$\begin{aligned} P(.4 \leq X \leq 1) &= F(1) - F(.4) \\ &= 1 - .4^2(3 - 2 \times .4) = .648 \end{aligned}$$

c)

$$\begin{aligned} P(X \leq .4 | X \leq .8) &= \frac{P(X \leq .4)}{P(X \leq .8)} \\ &= \frac{.4^2(3 - 2 \times .4)}{.8^2(3 - 2 \times .8)} = .3929 \end{aligned}$$

4.4. a) For $0 < x < 4$,

$$\begin{aligned} F(x) &= \int_0^x \frac{3}{64}u^2(4-u)du \\ &= \frac{3}{64} \int_0^x (4u^2 - u^3)du \\ &= \frac{3}{64} \left(\frac{4}{3}x^3 - \frac{1}{4}x^4 \right) \\ &= \frac{1}{256}x^3(16 - 3x). \end{aligned}$$

b)

$$\begin{aligned}P(X > 2) &= 1 - F(2) = 1 - \frac{1}{256} \times 2^3(16 - 6) \\ &= \frac{11}{16}.\end{aligned}$$

c)

$$\begin{aligned}P(X > 3) &= 1 - F(3) = 1 - \frac{1}{256} \times 3^3(16 - 9) \\ &= \frac{67}{256}.\end{aligned}$$

d)

$$\begin{aligned}1 - \frac{1}{256}x^3(16 - 3x) &= .1 \\ x^3(16 - 3x) &= 230.4 \\ x = 3.4298 &\quad \text{use solver}\end{aligned}$$