Math 323 Homework # 5

P158. 3.38. a)

$$P(Y \geq 2) = 1 - p(1) = 1 - (1 - .1)^0 .1 = .9$$

b)

$$\begin{aligned} P(Y > 2) &= 1 - p(1) - p(2) = 1 - (1-p)^0 p - (1-p)^1 p \\ &= 1 - p - (1-p)p = (1-p)^2. \end{aligned}$$

$$\begin{aligned} P(Y > 4) &= 1 - p(1) - p(2) - p(3) - p(4) = (1-p)^2 - (1-p)^2 p - (1-p)^3 p \\ &= (1-p)^3 - (1-p)^3 p = (1-p)^4. \end{aligned}$$

Thus

$$P(Y > 4 | Y > 2) = \frac{P(Y > 4)}{P > 2)} = \frac{(1-p)^4}{(1-p)^2} = (1-p)^2 = P(Y > 2).$$

P164. 3.52. a)

$$P(Y = 4) = .947 - .857 = .09 \qquad \text{(by table)}.$$

b)

$$P(Y < 4) = P(Y \leq 3) = .857$$

3.53. a) X is Poisson with $\lambda = 4$. Then

$$P(X = 0) = \frac{4^0}{0!} e^{-4} = e^{-4} = .0183$$

b)

$$P(X \geq 2) = 1 - P(X \leq 1) = 1 - .0183 - 4e^{-4} = .9085$$

c) Y is Poisson with $\lambda = 8$. Then

$$P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - .003 = .997 \qquad \text{(by table)}.$$

3.58. Let $X = \#$ of ignition system. Then X is Poisson with $\lambda = 3$

$$P(X \geq 4) = 1 - P(X \leq 3) = 1 - .647 = .353 \qquad \text{(by table)}.$$

3.63. Let $X = \#$ of imperfection for an 8-square-yard. Then, X is Poisson with $\lambda = 4 \times 8 = 32$. Note $C = 10X$. Then

$$\mathbb{E}\ (C) = 10\mathbb{E}\ (X) = 10 \times 32 = \$320$$

and

$$Var(C) = 10^2 Var(X) = 100 \times 32 = 3200$$
$$SD(C) = \sqrt{3200} = \$56.57$$

P170. 3.68. $X = \#$ of white balls selected. then, X is hypergeometric with $N = 7$, $k = 4$, $n = 2$.

a)

$$P(X = 1) = \frac{\binom{4}{1}\binom{3}{1}}{\binom{7}{2}} = \frac{4}{7}$$

b)

$$P(X \geq 1) = 1 - p(0) = 1 - \frac{\binom{4}{0}\binom{3}{2}}{\binom{7}{2}} = 1 - \frac{1}{7} = \frac{6}{7}$$

c)

$$P(X \geq 2 | X \geq 1) = \frac{P(X \geq 2)}{P(X \geq 1)} = \frac{\frac{\binom{4}{2}\binom{3}{0}}{\binom{7}{2}}}{\frac{6}{7}} = \frac{1}{3}$$

d)

$$\begin{aligned} P(W_2) &= P(W_2|W_1)P(W_1) + P(W_2|\bar{W}_1)P\bar{W}_1) \\ &= \frac{3}{6}\frac{4}{7} + \frac{4}{6}\frac{3}{7} = \frac{4}{7} \end{aligned}$$

3.70. $X = \#$ of defective. Then X is hypergeometric with $N = 10$, $k = 4$, $n = 5$.

$$\mathbb{E}\ (X) = \frac{4 \times 5}{10} = 2$$

and

$$Var(X) = 5 \times \frac{4}{10}\left(1 - \frac{4}{10}\right)\frac{10-5}{10-1} = \frac{2}{3}$$

Note that $C = 50X$. Then

$$\mathbb{E}\ (C) = 50 \times 2 = \$100$$

and

$$Var(C) = 50^2 \times \frac{2}{3} = 1666.67$$

Set $1 - \frac{1}{k^2} = .9$. Then $k = 3.16$ Take interval

$$\mu \pm 3.16\sigma = 100 \pm 3.16 \times \sqrt{1666.67} = 100 \pm 129.$$

With 90% confidence, it will below $229

3.75. $X = \#$ of misfiring ones removed. Then, X is hypergeometric with $N = 8$, $k = 2$, $n = 4$.

$$P(X = 2) = \frac{\binom{2}{2}\binom{6}{2}}{\binom{8}{4}} = \frac{3}{14}$$