Math 323 Homework # 4

P138, 3.11. Let X_i, $i = 1, 2$, be the net win. Then a)

$$\mathbb{E}\,(X_1) = (-1) \times \frac{1}{3} + 0 \times \frac{1}{3} + 1 \times \frac{1}{3} = 0,$$

$$Var(X_1) = (-1)^2 \times \frac{1}{3} + 0^2 \times \frac{1}{3} + 1^2 \times \frac{1}{3} = \frac{2}{3}.$$

b)

$$\mathbb{E}\,(X_2) = (-1) \times \frac{3}{5} + 0 \times \frac{1}{5} + 3 \times \frac{1}{5} = 0,$$

$$Var(X_2) = (-1)^2 \times \frac{3}{5} + 0^2 \times \frac{1}{5} + 3^2 \times \frac{1}{5} = \frac{12}{5}.$$

3.12. a)

$$\mathbb{E}\,(X) \approx 1\times.245+2\times.323+3\times.173+4\times.155+5\times.067+6\times.023+7\times.014 = 2.601$$

$$\mathbb{E}\,(X^2) \approx 1^2\times.245+2^2\times.323+3^2\times.173+4^2\times.155+5^2\times.067+6^2\times.023+7^2\times.014 = 8.763$$

$$Var(X) = \mathbb{E}\,(X^2) - (\mathbb{E}\,(X))^2 = 8.763 - 2.601^2 = 1.997799$$

$$SD(X) = \sqrt{1.997799} = 1.4134$$

b) $median = 2 < \mu$, skew to left. c)

$$\mu \pm \sigma = (1.188, 4.019), \qquad prob. = .323 + .173 + .155 = .651$$

$$\mu \pm 2\sigma = (-.2258, 4.019), \qquad prob = 1 - .023 - .014 = .963$$

Different comparing to .68 and .95

3.17. a)

$$1 - \frac{1}{k^2} = .9, \qquad k^2 = 10, \qquad k = \sqrt{10} = 3.16$$

$$\mu \pm 3.16\sigma = 4 \pm 3.16 \times .8 = (1.472, 6.528).$$

b) Yes, because the prob. is less than .1

P151. 3.21. a)

$$p(2) = \binom{4}{2} .2^2 (1 - .2)^2 = 6 \times .2^2 \times .8^2 = .1536$$

c)

$$\begin{aligned} P(X \leq 2) &= p(0) + p(1) + p(2) \\ &= \binom{4}{0} .2^0 .8^4 + \binom{4}{1} .2^1 .8^3 + .1536 \\ &= .9728 \end{aligned}$$

e)

$$Var(X) = np(1-p) = 4 \times .2 \times .8 = .64$$

3.22. a)

$$P(X \leq 6) = .25$$

c)

$$\begin{aligned} P(X = 8) &= P(X \leq 8) - P(X \leq 7) \\ &= .596 - .416 \\ &= .18 \end{aligned}$$

3.24. Let X be the number of survivors. Then, X is binomial with $n = 20$ and $p = .8$. a)

$$p(14) = P(X \leq 14) - P(X \leq 13) = .196 - .087 = .109$$

b)

$$P(X \geq 10) = 1 - P(X \leq 9) = 1 - .001 = .999$$

c)

$$P(X \leq 16) = .589$$

3.31. Let X be the number of radars which detects the aircraft. Then, X is binomial with $p = .9$ a)

$$P(X > 0) = 1 - p(0) = 1 - \binom{2}{0} .9^0 .1^2 = .99$$

b)

$$P(X > 0) = 1 - p(0) = 1 - \binom{4}{0} .9^0 .1^4 = .9999$$