Math 323 Homework # 3

P106. 2.35. Vehicles coming into an intersection can turn left or right or go straight ahead. Two vehicles enter an intersection in succession. Find the probability that at least one of the two vehicles turns left given that at least one of the two vehicles turns. What assumptions have you made?
Solution: Assume that the vehicles turn left or right or go straight ahead with equal probability and independent of each other.

Let

$$A = \{\text{at least one vehicles turns left}\}$$

and

$$B = \{\text{at least one vehicles turns}\}.$$

Then

$$\begin{aligned} P(B) &= P(T_1 \cup T_2) = 1 - P(\bar{T}_1 \bar{T}_2) \\ &= 1 - \frac{1}{3}\frac{1}{3} \\ &= \frac{8}{9} \end{aligned}$$

and

$$\begin{aligned} P(A) &= P(L_1 \cup L_2) = 1 - P(\bar{L}_1 \bar{L}_2) \\ &= 1 - \frac{2}{3}\frac{2}{3} \\ &= \frac{5}{9} \end{aligned}$$

Thus

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(A)}{P(B)} = \frac{5/9}{8/9} = \frac{5}{8}.$$

P106. 2.37. Solutions: a.

$$\frac{46263}{92911} = .50$$

b.

$$\frac{32949}{64053} = .51$$

c.

$$\frac{14738}{19801} = .74$$

d.

$$\frac{7067}{16065} = .44$$

e.

$$\frac{64053}{92911} = .69$$

P108. 2.45. 1)

$$P(AB) = P(A)P(B) = .9^2 = .81$$

2)

$$\begin{aligned} P(A \cup B) &= 1 - P(\bar{A}\bar{B}) = 1 - P(\bar{A})P(\bar{B}) \\ &= 1 - .1^2 = .99 \end{aligned}$$

2.47. Known:

$$P(F|A) = .2, \qquad P(F|B) = .1, \qquad P(A) = .7, \qquad P(B) = .3$$

Want to know: $P(A|F) =?$

As

$$P(AF) = P(F|A)P(A) = .2 \times .7 = .14$$

and

$$P(BF) = P(F|B)P(B) = .1 \times .3 = .03$$

we get

$$P(F) = .14 + .03 = .17$$

Thus

$$P(A|F) = \frac{P(AF)}{P(F)} = \frac{.14}{.17} = .8253$$

2.49. a) 4.7 means that 4.7% of the Aerobic Shoes sold are bought by persons under 14 years old.

67.0 means that 67% of the Walking Shoes sold are bought by Women.

b) .285 is the conditional probability that a Jogging/Running Shoes buyer is 25-34 years old.

c) Under 14 years old: $100,000 \times .346 = 34,600$

18-24-year old: $100,000 \times .097 = 9,700$

d) Not from this table because it does not provide the percentages of each type of shoes sold.

e) Answers may vary.

3.1. Possible values 0,1,2,3.

$$p(0) = \frac{\binom{4}{3}\binom{6}{0}}{\binom{10}{3}} = \frac{4}{\frac{10\times 9\times 8}{1\times 2\times 3}} = \frac{1}{30}$$

$$p(1) = \frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}} = \frac{\frac{4\times 3}{1\times 2}\times 6}{120} = \frac{3}{10}$$

$$p(2) = \frac{\binom{4}{1}\binom{6}{2}}{\binom{10}{3}} = \frac{4\times \frac{6\times 5}{1\times 2}}{120} = \frac{1}{2}$$

$$p(3) = \frac{\binom{4}{0}\binom{6}{3}}{\binom{10}{3}} = \frac{\frac{6\times 5\times 4}{1\times 2\times 3}}{120} = \frac{1}{6}$$

3.6. Possible values 0,1,2,3,4.

$$p(0) = (1-.4)^4 = .6^4 = .1298$$

$$p(1) = \binom{4}{1}.6^3 \times .4 = .3456$$

$$p(2) = \binom{4}{2}.6^2 \times .4^2 = .3456$$

$$p(3) = \binom{4}{3}.6^1 \times .4^3 = .1536$$

$$p(4) = .4^4 = .0256$$

3.8. Possible values 0,1,2,3,4.

$$p(0) = .7 \times .7 = .49$$

$$p(1) = 2 \times .7 \times .2 = .28$$

$$p(3) = 2 \times .1 \times .2 = .04$$

$$p(4) = .1^2 = .01$$

$$p(2) = 1 - .49 - .28 - .04 - .01 = .18$$

$$P(\text{at least } 1) = 1 - p(0) = 1 - .49 = .51$$