

Math 323 Homework # 13

8.2.

$$H_0 : \mu \geq 130 \text{ vs } H_a : \mu = 129$$

Reject  $H_0$  if

$$\frac{\bar{x} - 130}{2.1/\sqrt{40}} \leq -z_\alpha = -1.645.$$

Thus

$$\begin{aligned} \beta &= P\left(\frac{\bar{x} - 130}{2.1/\sqrt{40}} > -1.645 \mid \mu = 129\right) \\ &= P\left(\frac{\bar{x} - 129}{2.1/\sqrt{40}} > -1.645 + \frac{\sqrt{40}}{2.1} \mid \mu = 129\right) \\ &= P(Z \geq 1.367) = .5 - .4147 \\ &= .0853 \end{aligned}$$

8.7.  $n = 30$ ,  $\bar{x} = 6.8$ ,  $s = 0.9$

$$H_0 : \mu = 7 \text{ vs } H_a : \mu \neq 7, \quad \alpha = .05$$

Reject  $H_0$  if  $|Z| > z_{\alpha/2} = 1.96$

$$Z = \frac{\bar{x} - 7}{s/\sqrt{30}} = \frac{6.8 - 7}{.9/\sqrt{30}} = -1.2$$

Fail to reject  $H_0$ .

$$P\text{-value} = P(|Z| > 1.2) = 2(.5 - .3849) = .2303$$

8.11.

$$H_0 : \mu = 2500 \text{ vs } H_a : \mu < 2500$$

Reject  $H_0$  if  $T < -t_{.01}(5) = -3.365$

$$T = \frac{\bar{x} - 2500}{s/\sqrt{6}} = \frac{2418.33 - 2500}{79.35/\sqrt{6}} = -2.52$$

Fail to reject  $H_0$ .

8.24.  $n = 20$ ,  $s = 8$ ,  $\alpha = .05$

$$H_0 : \sigma = 5 \text{ vs } H_a : \sigma > 5$$

Reject  $H_0$  if  $U > \chi_{.05}^2 = 30.1435$

$$U = \frac{(n-1)s^2}{5^2} = \frac{19 \times 8^2}{5^2} = 48.64$$

Reject  $H_0$ .

8.47.

$$H_0: p_1 = .5, p_2 = .2, p_3 = .2, p_4 = .1$$

Reject  $H_0$  if  $X^2 > \chi_{.05}^2(3) = 7.81473$

$$\begin{aligned} X^2 &= \frac{(48 - 100 \times .5)^2}{100 \times .5} + \frac{(18 - 100 \times .2)^2}{100 \times .2} + \frac{(21 - 100 \times .2)^2}{100 \times .2} + \frac{(13 - 100 \times .1)^2}{100 \times .1} \\ &= 1.23 \end{aligned}$$

Fail to reject  $H_0$ .

8.53. a)  $H_0$ : independent

Reject  $H_0$  if  $X^2 > \chi_{.05}^2(2) = 5.99147$

$$X_1 = 426 = X_2.$$

$$\begin{aligned} X^2 &= \frac{\left(290 - \frac{528}{2}\right)^2}{\frac{528}{2}} + \frac{\left(94 - \frac{219}{2}\right)^2}{\frac{219}{2}} + \frac{\left(42 - \frac{105}{2}\right)^2}{\frac{105}{2}} \\ &= \frac{\left(238 - \frac{528}{2}\right)^2}{\frac{528}{2}} + \frac{\left(125 - \frac{219}{2}\right)^2}{\frac{219}{2}} + \frac{\left(63 - \frac{105}{2}\right)^2}{\frac{105}{2}} \\ &= 13.709 \end{aligned}$$

Reject  $H_0$

8.61.

$$H_0: p_1 = p_2$$

Reject  $H_0$  if  $X^2 > \chi_{.05}^2(1) = 6.6349$

$$\hat{p} = \frac{600 + 500}{2000} = .55$$

$$X^2 = \frac{(600 - 1000 \times .55)^2}{1000 \times .55 \times .45} + \frac{(500 - 1000 \times .55)^2}{1000 \times .55 \times .45} = 20.20$$

Reject  $H_0$