Math 323 Homework # 12

6.40. $\sigma^2 \approx 50$, $n = 15$
a)

$$\begin{aligned} P(S^2 \geq 80) &= P\left(\frac{(n-1)S^2}{\sigma^2} \geq \frac{14 \times 80}{50}\right) \\ &= P(U \geq 22.4) \end{aligned}$$

is between .05 and .1
b)

$$\begin{aligned} P(S^2 \leq 20) &= P\left(U \leq \frac{14 \times 20}{50}\right) = P(U \leq 5.6) \\ &= 1 - P(U > 5.6) \approx 1 - .975 = .025 \end{aligned}$$

c) Let k be such that

$$1 - \frac{1}{k^2} = .75$$

Thus, $k = 2$. Note that

$$\mathbb{E}\ (S^2) = 50, \qquad SD(S^2) = \sqrt{\frac{2 \times 50^2}{14}} = 18.898$$

The interval is

$$50 \pm 2 \times 18.898 = (12.204, 87.796)$$

d) Normal population
6.42. $\mu = 80$, $\sigma = 10$, $n = 12$

$$P(S > 15) = P\left(U > \frac{11 \times 15^2}{10^2}\right) = P(U > 24.75) \approx .01$$

7.1. a) All of them are unbiased because

$$\mathbb{E}\ \hat{\theta}_i = \theta, \qquad i = 1, 2, 3, 4.$$

b)

$$V(\hat{\theta}_1) = \sigma^2 = \theta^2$$

$$V(\hat{\theta}_2) = \frac{1}{4} \times 2\theta^2 = \frac{1}{2}\theta^2$$

$$V(\hat{\theta}_3) = \frac{1}{9}(1+4)\theta^2 = \frac{5}{9}\theta^2$$
$$V(\hat{\theta}_4) = \frac{1}{3}\theta^2$$

Thus, the best is $\hat{\theta}_4$

7.10. $n = 40, \quad \bar{x} = 46, \quad s = 3, \quad 1 - \alpha = .95$

$$\bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}} = 46 \pm 1.96 \times \frac{3}{\sqrt{40}} = 46 \pm .93 = (45.07, 46.93)$$

7.12. $\sigma = 18, \quad B = 4, \quad 1 - \alpha = .9$ Then, $z_{.05} = 1.645$

$$n = \left(\frac{z_{\alpha/2}\sigma}{B}\right)^2 = \left(\frac{1.645 \times 18}{4}\right)^2 \approx 55$$

7.20. $\bar{x} = 9, \quad s = 6.424$

$$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} = 9 \pm 1.796 \times \frac{6.424}{\sqrt{12}} = 9 \pm 3.33 = (5.67, 12.33)$$

7.24.

$$\left(\frac{(n-1) \times s^2}{\chi^2_{.025}}, \frac{(n-1) \times s^2}{\chi^2_{.975}}\right) = \left(\frac{11 \times 6.424^2}{21.92}, \frac{11 \times 6.424^2}{3.81575}\right) = (20.71, 118.85)$$

7.66.

$$\begin{aligned} L(\lambda) &= \frac{\lambda^{x_1}}{x_1!}e^{-\alpha}\frac{\lambda^{x_2}}{x_2!}e^{-\alpha} \cdots \frac{\lambda^{x_n}}{x_n!}e^{-\alpha} \\ &= \lambda^{x_1+x_2+\cdots+x_n}e^{-n\lambda}\frac{1}{x_1!x_2!\cdots x_n!} \end{aligned}$$

$$\ln L(\lambda) = (x_1 + x_2 + \cdots + x_n)\ln\lambda - n\lambda - \ln(x_1!x_2!\cdots x_n!)$$
$$(\ln L(\lambda))' = \frac{x_1 + x_2 + \cdots + x_n}{\lambda} - n = 0$$
$$\hat{\lambda} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$