

Math 323 Homework # 11

6.1. b)

$$\bar{x} = 1.6633, \quad s^2 = .228097 \text{ use calculator}$$

c) Use Tchebysheff inequality

$$P(|X - \mu| < k\sigma) \geq 1 - \frac{1}{k^2} = .75$$

$$\frac{1}{k^2} = .25, \quad k = 2$$

$$\begin{aligned} k \pm 2\sigma &\approx \bar{x} \pm 2s \\ &= 1.6633 \pm 2\sqrt{.228097} \\ &= (.7081, 2.6185) \end{aligned}$$

6.16. $\mu = 4, \sigma = .8$

a)

$$\begin{aligned} P(1 < \bar{x} < 5) &= P\left(\frac{1-4}{.8/\sqrt{30}} < Z < \frac{5-4}{.8/\sqrt{30}}\right) \\ &= P(-20.54 < Z < 6.85) = 1 \end{aligned}$$

b)

$$\begin{aligned} P(X_1 + \cdots + X_{30} < 115) &= P\left(\bar{x} < \frac{115}{30}\right) \\ &= P\left(Z < \frac{\frac{115}{30} - 4}{.8/\sqrt{30}}\right) \\ &= P(Z < -1.14) \\ &= .5 - .3729 = .1271 \end{aligned}$$

6.24. midian=33

Let X be the # of people in the sample younger than 33. Then, X is binomial with $p = .5$ and $n = 100$.

$$\mu = 100 \times .5 = 50, \quad \sigma = \sqrt{100 \times .5 \times .5} = 5$$

$$\mu \pm 2\sigma = 50 \pm 10 = (40, 60) \text{ in } (0, 100)$$

n is large.

$$\begin{aligned} P(X \geq 60) &\stackrel{CC}{=} P(X \geq 59.5) \\ &= P\left(Z \geq \frac{59.5 - 50}{5}\right) \\ &= P(Z \geq 1.9) = .5 - .4713 = .0287 \end{aligned}$$

6.30. a) Let X be the number of right turns. Then, X is binomial with $n = 500$, $p = \frac{1}{3}$

$$\mu = \frac{500}{3} = 166.7, \quad \sigma = \sqrt{500 \times \frac{1}{3} \times \frac{2}{3}} = 10.54$$

$$\mu \pm 2\sigma = (145.6, 187.7) \text{ in } (0, 500)$$

n is large.

$$\begin{aligned} P(X \leq 150) &\stackrel{CC}{=} P(X \leq 150.5) = P\left(Z \leq \frac{150.5 - 166.7}{10.54}\right) \\ &= P(Z \leq -1.53) = .5 - .437 = .063 \end{aligned}$$

b) Y the number of turns, Y is binomial $n = 500$, $p = \frac{2}{3}$. Then

$$\begin{aligned} P(Y \geq 350) &= P(X \leq 149) = P(X \leq 149.5) \\ &= P\left(Z \leq \frac{149.5 - 166.7}{10.54}\right) = P(Z \leq -1.63) \\ &= .5 - .4484 = .0516 \end{aligned}$$