Math 323 Homework # 10

5.42. a)

$$f_1(x_1) = \int_0^{x_1} 3x_1 dx_2 = 3x_1^2, \qquad 0 < x_1 < 1$$

$$f_2(x_2) = \int_{x_2}^1 3x_1 dx_1 = \frac{3}{2}x_1^2|_{x_2}^1 = \frac{3}{2}(1 - x_2^2), \qquad 0 < x_2 < 1$$

b)

$$\begin{aligned} P\left(X_1 \le \frac{3}{4}, X_2 \le \frac{1}{2}\right) &= \int_0^{\frac{1}{2}} \int_{x_2}^{\frac{3}{4}} 3x_1 dx_1 dx_2 \\ &= \int_0^{\frac{1}{2}} \frac{3}{2}x_1^2|_{x_2}^{\frac{3}{4}} dx_2 \\ &= \frac{3}{2}\int_0^{\frac{1}{2}} \left(\left(\frac{3}{4}\right)^2 - x_2^2\right) dx_2 \\ &= \frac{3}{2}\left(\frac{9}{16}\cdot\frac{1}{2} - \frac{1}{3}\cdot\frac{1}{8}\right) \\ &= \frac{23}{64} \end{aligned}$$

c) Since $X_1 \ge X_2$, we get

$$P\left(X_1 \le \frac{1}{2}\middle| X_2 \ge \frac{3}{4}\right) = 0.$$

5.45. a)

$$f(x_1|x_2) = \frac{3x_1}{\frac{3}{2}(1 - x_2^2)} = \frac{2x_1}{1 - x_2^2}, \qquad x_2 < x_1 < 1$$

b)

$$f(x_2|x_1) = \frac{3x_1}{3x_1^2} = \frac{1}{x_1}, \qquad 0 < x_2 < x_1$$

c) Since

$$f_1(.5)f_2(.6) > 0 = f(.5, .6)$$

they are indep.

d)

$$\begin{aligned} P\left(X_1 \le \frac{3}{4}\middle| X_2 = \frac{1}{2}\right) &= \int_{\frac{1}{2}}^{\frac{3}{4}} \frac{2x_1}{1 - \frac{1}{4}} dx_1 \\ &= \frac{4}{3}x_1^2\Big|_{\frac{1}{2}}^{\frac{3}{4}} = \frac{5}{12} \end{aligned}$$

5.60. a)

$$\mathbb{E}\ (X_2|X_1 = x_1) = \int_0^{x_1} \frac{1}{x_1} x_2 dx_2 = \frac{x_1}{2}$$

b)

$$\mathbb{E}\ (X_2) = \mathbb{E}\ \ (X_1/2) = \int_0^1 \frac{x_1}{2} 3x_1^2 dx_1 = \frac{3}{8} x_1^4|_0^1 = \frac{3}{8}$$

c)

$$\mathbb{E}\ (X_2) = \int_0^1 x_2 \frac{3}{2}(1 - x_2^2) dx_2 = \frac{3}{2}\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{3}{8}.$$