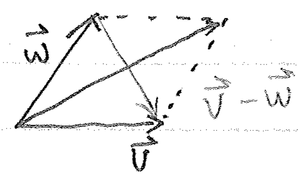
Review of ch 3

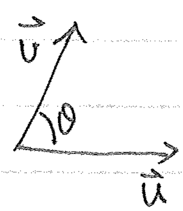
1. $\vec{v}+\vec{w}$



2. If $\vec{v}=(v_1, v_2, v_3)$, then $\vec{v}+\vec{w}=(v_1+w_1,\ v_2+w_2,\ v_3+w_3)$
$\vec{w}=(w_1, w_2, w_3)$

$\|\vec{v}\| = \sqrt{v_1^2+v_2^2+v_3^2}$

3. $\vec{u}\cdot\vec{v} = \|\vec{u}\|\,\|\vec{v}\|\cos\theta$



- $\vec{u}\cdot\vec{v} = u_1v_1+u_2v_2+u_3v_3$
- $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|}$

- $\vec{u},\vec{v}$ orthogonal $\Leftrightarrow$ $\vec{u}\cdot\vec{v}=0$
- $\mathrm{Proj}_{\vec{a}}\,\vec{u} = \dfrac{\vec{u}\cdot\vec{a}}{\|\vec{a}\|^2}\,\vec{a}$

Review of Ch 4

1. R^n consists of n-tuples $\vec{u} = (u_1, u_2, \dots, u_n)$
   - $\vec{u} + \vec{v} = (u_1 + v_1, \dots, u_n + v_n)$
   - $k\vec{u} = (ku_1, \dots, ku_n)$
   - $\vec{u} \cdot \vec{v} = u_1 v_1 + \cdots + u_n v_n$ inner product
   - $\|\vec{u}\| = \sqrt{u_1^2 + \cdots + u_n^2}$
2. Cauchy-Schwarz inequality

   $\|\vec{u} \cdot \vec{v}\| \le \|\vec{u}\| \, \|\vec{v}\|$
3. Triangle inequality

   $\|\vec{u} + \vec{v}\| \le \|\vec{u}\| + \|\vec{v}\|$
4. $\vec{u} \cdot \vec{v} = \frac{1}{4}\|\vec{u} + \vec{v}\|^2 - \frac{1}{4}\|\vec{u} - \vec{v}\|^2$
5. $\vec{u}, \vec{v}$ orthogonal $\Leftrightarrow$ $\vec{u} \cdot \vec{v} = 0$
6. $T: R^n \to R^m$ ~~$T(x_1, \dots, x_n) = (w_1, \dots, w_m)$~~

   linear transformation

   $\Leftrightarrow$ $T(\vec{x}) = A\vec{x}$, A $m \times n$ matrix

   $A = [T]$, $T = T_A$
7. If $T_A : R^n \to R^k$ and $T_B : R^k \to R^m$, then $T_B \circ T_A : R^n \to R^m$

   $T_B \circ T_A = T_{BA}$
8. $T: R^n \to R^m$ 1-to-1 if $T(\vec{x}) \neq T(\vec{y})$ when $\vec{x} \neq \vec{y}$
9. $T = T_A : R^n \to R^n$ 1-to-1

   $\Leftrightarrow$ A is invertible

   $\Leftrightarrow$ T_A has range R^n

   $T_A^{-1} = T_{A^{-1}}$
10. $[T] = \left[T(\vec{e}_1) \;\; T(\vec{e}_2) \; \cdots \; T(e_n) \right]$

Review of Ch5 (5.1 – 5.4)

1. V is vector space if addition and scalar multiplication are defined and satisfy axioms (1)—(10)
2. W is a subspace of V if it is a subset of V and is a space itself
   - $\vec{u}, \vec{v}$ in $W \Rightarrow \vec{u}+\vec{v}$ in W
   - $\vec{u}$ in W, k real $\Rightarrow k\vec{u}$ in W
3. Solution set of $A\vec{x}=\vec{0}$ is a subspace of R^n
4. $\vec{v}$ is a linear combination of $\vec{v}_1, \cdots, \vec{v}_r$ if
   $$\vec{v} = k_1\vec{v}_1 + k_2\vec{v}_2 + \cdots + k_r\vec{v}_r$$
   - All linear combinations of $\vec{v}_1, \cdots, \vec{v}_r$ is a subspace spanned by $\vec{v}_1, \cdots, \vec{v}_r$
5. $\{\vec{v}_1, \cdots, \vec{v}_r\}$ is linearly independent if
   $$k_1\vec{v}_1 + \cdots + k_r\vec{v}_r = \vec{0} \quad \text{implies} \quad k_1 = \cdots = k_r = 0$$
   - $S=\{\vec{v}_1, \cdots, \vec{v}_r\}$ in R^n, $r>n \Rightarrow S$ is linearly dependent
6. $f_1, \cdots, f_n$ real functions, they are linearly independent if $W(x) \neq 0$ for some x
7. $S=\{\vec{v}_1, \cdots, \vec{v}_n\}$ is a basis if it is linearly independent and $\mathrm{span}(S)=V$
   - $(\vec{v})_S = (c_1, \cdots, c_n)$ if $\vec{v} = c_1\vec{v}_1 + \cdots + c_n\vec{v}_n$
   - The dimension of V is $\dim(V)=n$ (all bases have the same number of vectors)
   - If $\dim(V)=n$ and $S=\{\vec{v}_1, \cdots, \vec{v}_n\}$ is ~~a basis~~, linearly independent, then S is a basis.