Review of ch 1

1. Linear system vs augmented matrix
\[ a_1 x_1 + \cdots + a_n x_n = b_1 \]
\[ \vdots \]
\[ a_m x_1 + \cdots + a_n x_n = b_m \]
\[
\begin{bmatrix}
  a_{11} & \cdots & a_{1n} & b_1 \\
  \vdots & \ddots & \vdots & \vdots \\
  a_{m1} & \cdots & a_{mn} & b_m \\
\end{bmatrix}
\]

2. Solve linear system by elementary row operations
   - Multiply a row by a non-zero constant
   - Interchange two rows
   - Add a multiple of one row to another row

3. Row-echelon form
   - The 1st non-zero element in each row is 1;
   - The rows of zeros are at the bottom
   - The leading 1 in the lower row occurs farther to the right than the leading 1 in the higher row
   - Reduced row echelon form if, in addition
     - Each column that contains a leading 1 has zero everywhere else

4. A linear system has either
   - Exactly one solution
   - Infinite many solutions (consistent free-variable)
   - No solution

5. Homogeneous linear system
\[ a_1 x_1 + \cdots + a_n x_n = 0 \]
\[ \vdots \]
\[ a_m x_1 + \cdots + a_n x_n = 0 \]
has at least one sol. \((x_1 = \cdots = x_n, \text{ trivial sol.})\)

6. Definition of \(A+B, kA, AB, A^T, \text{tr}(A)\)

7. Linear system \(Ax = b\)
8. In general, $AB \neq BA$

9. $O_{m \times n}$ = zero matrix of size $m \times n$
   $I_n$ = identity matrix of size $n \times n$
   $A + 0 = O + A = A$
   $A \cdot 0 = 0 \cdot A = 0$
   $A \cdot I = I \cdot A = A$

10. $A$ is invertible if there is $B$ such that $AB = BA = I$
    $B = A^{-1}$ is the inverse of $A$

11. $A, B$ both $n \times n$, thus $AB$ is invertible $\iff$ $A, B$ both invertible.
    $(AB)^{-1} = B^{-1}A^{-1}$

12. $(AB)^T = B^TA^T$, $(A^T)^{-1} = (A^{-1})^T$

13. Elementary matrices: obtained from $I$ by a row operation.
    - Row operation on $A \iff$ Multiple $A$ by the elementary matrix

14. How to find $A^{-1}$?
    - Put $A, I$ together, reduce $A$ to $I$. Then $I$ is reduced to $A^{-1}$.

15. The following are equiv.
   a) $A$ invertible
   b) $Ax = \vec{0}$ only the trivial sol.
   c) Reduced echelon form of $A$ is $I$
   d) $A = E_1 \cdots E_k$
   e) $Ax = \vec{b}$ consistent for each $\vec{b}$
   f) $Ax = \vec{b}$ exactly one sol. $\vec{x} = A^{-1}\vec{b}$

16. Diagonal, triangular, symmetric matrices

17. $A$ has an LU decomposition if only types 1 & 3 operations needed reducing $A$ to echelon form
$U = \text{echelon form}$

$L$ diagonal: reciprocal of the multiplier of type $I$

below diagonal: negative of the multiplier of type $III$
Review of ch 2

1. A \( n \times n \). The \( \det(A) \) is the sum of all signed elementary product

\[
\det(A) = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix}
\]

2. \[
\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}
\]

\[
\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{11}a_{23}a_{32}
\]

3. If \( A \) is triangular, then \( \det(A) = a_{11}a_{22} \cdots a_{nn} \)

4. \( B \) obtained from \( A \) by row operation
   - type I with multiplier \( k \), \( \det(B) = k \det(A) \)
   - type II, \( \det(B) = -\det(A) \)
   - type III, \( \det(B) = \det(A) \)

5. If \( A \) has two proportional rows (columns), \( \det(A)=0 \)

6. \( A \) invertible \( \iff \) \( \det(A) \neq 0 \)

7. \( A, B, C \ n \times n \), differ only in \( r \)th row,
   \( r \)th row of \( C \) is the sum of \( r \)th rows of \( A \) and \( B \)
   then \( \det(C) = \det(A) + \det(B) \)

8. \( \det(ka) = k^n \det(A) \)
   \( \det(AB) = \det(A) \det(B) \)
   \( \det(A^{-1}) = \frac{1}{\det(A)} \)

9. \( \lambda \) is an eigenvalue of \( A \) if \( A\vec{x} = \lambda \vec{x} \)

   \( \det(A - \lambda I) = 0 \)

10. \( C_{ij} = (-1)^{i+j} M_{ij} \) is the cofactor of \( a_{ij} \)
    \( M_{ij} \) is the minor of \( a_{ij} \)
\[ \det(A) = a_{11}C_{11} + \cdots + a_{nn}C_{nn} \]
\[ \det(A) = a_{ij}c_{ij} + \cdots + a_{nj}c_{nj} \]

11. \[ \text{adj}(A) = (c_{ij})_{n \times n} \]
\[ A^{-1} = \frac{1}{\det(A)} \text{adj}(A) \]

12. If \( A \) is invertible, then \( Ax = \bar{b} \) has a unique solution given by
\[ x_1 = \frac{\det(A_1)}{\det(A)}, \ldots, x_n = \frac{\det(A_n)}{\det(A)} \]
\[ \text{adj}(A) = A_j; \text{ the matrix obtained by replacing the } j\text{th column of } A \text{ by } \bar{b} \]