Math 241 **Computing Line Integrals**

Work = $\int_C \mathbf{F} \bullet \mathbf{dr}$ where $\mathbf{F(x,y,z)} = < \mathbf{P}(x,y,z), \mathbf{Q}(x,y,z), \mathbf{R}(x,y,z) >$

First rewrite in the form $\int_C \mathbf{F} \bullet \mathbf{dr} = \int_C \mathbf{Pdx} + \mathbf{Qdy} + \mathbf{Rdz}$

Then parametrize C, check orientation, substitute and evaluate.

Example 1: $\mathbf{F(x,y,z)} = < \mathbf{x}, \mathbf{xy}, \mathbf{x} + \mathbf{z} >$ and C is a line from P(0, 1, 2) to Q(1, 0, 4)

Rewrite $\int_C \mathbf{F} \bullet \mathbf{dr} = \int_C x\mathbf{dx} + xy\mathbf{dy} + (x+z)\mathbf{dz}$

C: the vector PQ is <1, -1, 2>

x = t dx = dt $\int_C \mathbf{F} \bullet \mathbf{dr} = \int_0^1 (t)(1) + (t)(-t+1)(-1) + (t+2t+2)2\,dt$

y= -t + 1 dy = -1 dt $= \int_0^1 t^2 + 7t + 4\,dt$

z = 2t + 2 dz = 2 dt $= 47/6$

$0 \leq t \leq 1$

Example 2: Change the curve to a helix

x = cos t dx = -sin t dt $\int_C \mathbf{F} \bullet \mathbf{dr} = \int_0^{2\pi} (\cos t)(-\sin t) + (\cos t \sin t)(\cos t) + (\cos t + t)\,dt$

y= sin t dy = cos t dt $\int_C \mathbf{F} \bullet \mathbf{dr} = \int_0^{2\pi} -\cos t \sin t + \cos^2 t \sin t + \cos t + t\,dt$

z = t dz = dt $= 2\pi^2$

$0 < t < 2\pi$

Example 3: C is the curve C_1: $< t, t^2, t^3 >$ $0 \leq t \leq 1$, followed by C_2, the line from P (1, 1, 1) to Q (1, 2, 3) This is really two separate computations which are added together. The notation for C is C = $C_1 + C_2$ and $\int_{C_1+C_2} \mathbf{F} \bullet \mathbf{dr} = \int_{C_1} \mathbf{F} \bullet \mathbf{dr} + \int_{C_2} \mathbf{F} \bullet \mathbf{dr}$

C_1:

x = t dx = dt $\int_C \mathbf{F} \bullet \mathbf{dr} = \int_0^1 t + t t^2\, 2t + (t + t^3)3t^2 dt$

y = t^2 dy = 2t dt $= \int_0^1 3t^5 + 2t^4 + 3t^3 + t\,dt$

z = t^3 dz = $3t^2$ dt $= 1/2 + 2/5 + 3/4 + 1/2 = 43/20$

C_2: the direction vector PQ is <1, 1, 2>

x = t + 1 dx = dt $\int_C \mathbf{F} \bullet \mathbf{dr} = \int_0^1 (t+1) + (t+1)(t+1) + (t+1+2t+1)2dt$

y = t + 1 dy = dt $= \int_0^1 t^2 + 9t + 6dt$

z = 2t+ 1 dz = 2 dt $= 1/3 + 9/2 + 6 = 65/6$

$\int_{C_1+C_2} \mathbf{F} \bullet \mathbf{dr} = \int_{C_1} \mathbf{F} \bullet \mathbf{dr} + \int_{C_2} \mathbf{F} \bullet \mathbf{dr} = 43/20 + 65/6 = 779/60$