Math 241 Supplementary Problems

1. Memorize this. $\int_0^1 x^n \, dx =$

2. Memorize this. $\int_0^{\pi/2} \sin x \, dx = \int_0^{\pi/2} \cos x \, dx =$

3. $\int_0^{2\pi} \sin^{23}(345\theta)\cos^{12}(345\theta)\, d\theta$

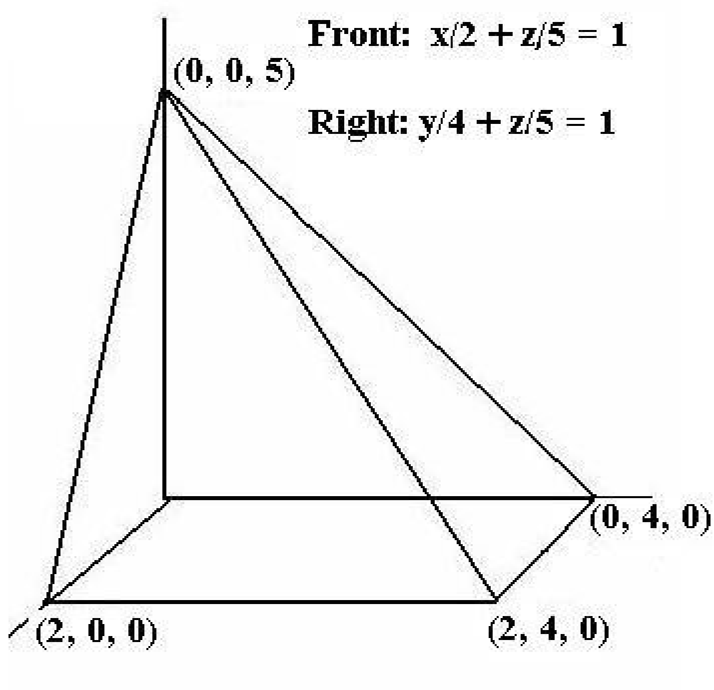
4. $\int_0^{\pi} \sin^2\theta \, d\theta = \int_0^{\pi} \cos^2\theta \, d\theta$ Hint: Add these two integrals together first.

5. $\int_0^{2\pi} \sin^{247}\theta \, d\theta$

6. Given the three points P Q and R determine all values of z that make triangle PQR a right triangle. P(8, -1, 4); Q(-1, 2, 1); R(2, 3, z).

7. Let R be the region bounded by a quarter circle of radius a in the first quadrant with constant density k. Find the center of mass and the moments of inertia I_x and I_y.

8. Write out the triple integral for the volume of the solid shown in all six possible orders. Evaluate at least 2 of these integrals. Hint: the dzdydx and dzdxdy orders are the trickiest. Do these last.



9. Find the moment of inertia of a homogeneous sphere of radius a about a diameter.

10 The one dimensional heat equation is: $u_t = a^2 u_{xx}$, where the temperature u(x, t) depends on the position on thwe x-axis and the time t.

a) Verify that the function $u_n(x,t) = e^{-a^2 n^2 t}\sin(nx)$ solves the heat equation with boundary conditions (B.C.) u(0,t) = u(π,t) = 0 and initial condition (I.C.) u(x,0) = sin(nx).

b) Suppose the initial conditions are changed to (I.C.) $u(x,0) = \sum_{n=1}^{k} b_n \sin(nx)$, for some constants b_n Show that the solution is now: $u(x,t) = \sum_{n=1}^{k} b_n u_n(x,t) = \sum_{n=1}^{k} b_n e^{-a^2 n^2 t}\sin(nx)$

Then the question arises, "exactly which functions can be expressed as a sum of sine functions?" The attempts to answer to this question lead to the theory of Fourier series and much of modern mathematics.

11. Find the distance from the point P(5,3) to the line 2x - 6y = 7.

12. Find the plane determined by P, Q, and R. P(1, -1, 3), Q(2, 1 4), R(0, 1, 0).

13. Find the distance from the point P(2, 3, -4) to the plane x + 2y - z = 5

14. Find the distance from the point P(2, 3, 0) to the line **r**(t) = < 3t, 5t + 1, -2t + 3 >.

15. Find the distance between the lines **r**(t) = < 3t, 5t + 1, -2t + 3 > and **l**(t) = < t + 1, t, t + 4 >.

16. Find the intersection of the plane 2x - y + z = 4 and the line through P and Q.
P(1, 2, 3) and Q(6,-2,0)

17. Do the line **r**(t) = < 3t, 5t + 1, -2t + 3 > and the plane x + 2y - z = 4 intersect or are they parallel? If they are parallel find the distance between them, if they intersect, find the intersection point.