Math 241 **Surface Integrals**

	Surface equation	Domain D	$\mathbf{N_S}$
Explicit	z = f(x, y)	in xy-plane	$\langle f_x, f_y, -1 \rangle$
	y = f(x, z)	in xz-plane	$\langle f_x, -1, f_z \rangle$
	x = f(y, z)	in yz-plane	$\langle -1, f_y, f_z \rangle$
Implicit	G(x, y, z) = c		
		in xy-plane	$\nabla G / \nabla G \cdot \hat{k}$
		in xz-plane	$\nabla G / \nabla G \cdot \hat{j}$
		in yz-plane	$\nabla G / \nabla G \cdot \hat{i}$
Parametric	**r**(u, v)	in uv-plane	$r_u \times r_v$

Surface Area: $\iint_S 1\, dS = \iint_D |\mathbf{N}_\mathrm{s}|\, dA$

Scalar Surface integral: $\iint_S h(x,y,z)\, dS = \iint_D h\, |\mathbf{N}_\mathrm{s}|\, dA$

The variables need to be expressed in terms of the two independent variables. For example, in the first explicit case

$$\iint_S h(x,y,z)\, dS = \iint_D h(x,y,f(x,y))\, |\mathbf{N}_\mathrm{s}|\, dxdy$$

Flux of **F** through S.

The surface S must be oriented here, so $\mathbf{N_S}$ may have to have its sign reversed.

Flux = $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iint_D \mathbf{F} \cdot \mathbf{N}_\mathrm{s}\, dA$

As above, **F**(x, y, z) needs to be expressed in terms of the independent variables. For example, in the third explicit case, you would have

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iint_D \mathbf{F}(f(y,z),y,z) \cdot \mathbf{N}_\mathrm{s}\, dydz$$