The sequence of harmonic numbers $H_n$ are defined by: $H_n = \sum_{k=1}^{n} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$

The first few harmonic numbers are: $H_1 = 1; H_2 = 3/2; H_3 = 11/6$

1. Find the smallest values of $n$ that makes $H_n \geq 2; \ H_n \geq 3; \ H_n \geq 4$. Just use a calculator. (extra credit: find the smallest $n$ for $H_n \geq 9$) (extra extra credit: find an explicit formula for $H_n$. You could publish the result.)

We want to estimate how fast this sequence grows by comparing it to the area under $y = 1/x$.

2. Draw pictures of $R_6$ and $L_6$ (the left and right hand Riemann sums) for $y = 1/x$ on $[1, 7]$. Note that $\Delta x$ is 1 here. Use them to show that $H_7 - 1 < \ln 7 < H_6$.

3. Show that $H_n - 1 < H_{n+1} - 1 < \ln(n + 1) < H_n$

4. Evaluate $\lim_{n \to \infty} H_n$

5. If the terms of the harmonic sequence were computed at a rate of one per nano-second since the beginning of the universe (about $13.8 \times 10^9$ years ago) about how large would $H_n$ be? (It would probably take too long to compute this exactly, so use the estimate from 3).