Math 142  Pre-test 1

Please show your work. There are 75 points on this test.

1. (12 pts.) The acceleration of a particle along the x-axis is given by
   \(a(t) = (t - 1) \text{ m/sec}^2\) (t is in seconds.) If the particle starts at \(x = 0\) when \(t = 0\) and has
   initial velocity \(v(0) = -3 \text{ m/sec}\)
   a) Determine the position function \(f(t)\).
   b) Does the particle ever reach the origin again? If so, when?

2. (8 pts.) Find the area above the x-axis and under \(y = x - x^3\) on the interval \([0, 1]\). Sketch
   the region.

3. (42 pts.) Here are 8 integrals to evaluate. Do only six of them. Check by
   differentiating. If your answer is incorrect, indicate this and try again. Don’t erase your
   first attempt! Circle the 6 problems you want graded.

   a. \(\int \cos^6(3x) \sin(3x) \, dx\)  
   b. \(\int \frac{(\ln x)^3}{x} \, dx\)  
   c. \(\int x(3x^2 - x) \, dx\)

   d. \(\int e^{\sec x} \sec x \tan x \, dx\)  
   e. \(\int_2^9 \frac{1}{\sqrt{3x - 2}} \, dx\)  
   f. \(\int_0^\infty \frac{e^{2x}}{e^{3x} + 5} \, dx\)

   g. \(\int_0^1 (5 - x^3)^{2014} \, dx\)  
   h. \(\int_0^{10} e^{-x} \, dx\)
4. (8 pts.) Let \( g(x) = \int_0^x f(t) \, dt \) where the graph of \( f(t) \) is shown below
   
   a. Determine \( g(0), g(1), g(3) \) and \( g(4) \).
   b. Sketch a graph of \( g(x) \) on \([0, 4]\).

5. (5 pts.) Estimate the sizes of the numbers below well enough to determine the order of the four quantities. Say briefly how you are getting these estimates. Then list them in order, from smallest to largest. The value of \( n \) is your student id

   Here is the graph of \( f(x) \) on \([0, 3]\):

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<th>Smallest</th>
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<th>Largest</th>
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   a. \( R_3 = \sum_{k=1}^{3} f(k) \)
   b. \( R_n = \sum_{k=1}^{n} f\left(\frac{3k}{n}\right) \cdot \frac{3}{n} \)
   c. \( \sum_{k=1}^{n} \frac{2}{k} \)
   d. \( \int_0^n e^{x} \, dx \)