Math 142 **Geometric Series**

A geometric series has this general form: $a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^k + \cdots$ Each successive term is r times the preceding term.

For example:

$1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots$ a = 1 and r = ½ .

$1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \frac{1}{2^4} + \cdots$ a = 1 and r = -½

$3 - 3 + 3 - 3 + 3 - 3 + \cdots$ a = ____ and r = ____

$1 + (1.1) + (1.1)^2 + (1.1)^3 + (1.1)^4 + \cdots$ = 1 + 1.1 + 1.21 + 1.331 + 1.4641 + . . . a = ____ and r = ____

The repeating decimal $.33333\cdots = .3 + .03 + .003 + .0003 + \cdots$ a = ____ and r = ____

We want to determine a way to sum up the entire geometric series. The idea is to get a formula for adding up finitely many terms and then taking a limit. Let $S_k = a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^k$ be the sum of the the first k terms. It is called the kth partial sum. If each term is multiplied by r, it almost looks like the the sum of k+1 terms, so if we subtract the two quantities, most terms cancel.

$$S_k = a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^k$$
$$rS_k = \quad ar + ar^2 + ar^3 + ar^4 + \cdots + ar^k + ar^{k+1}$$

Subtracting gives $S_k - rS_k = S_k(1-r) = a - ar^{k+1}$ and dividing by (1- r) gives the formula:

$S_k = \dfrac{a(1-r^{k+1})}{(1-r)}$ for a finite geometric series, as long as r ≠ 1.

For the **infinite** series, we compute $\lim_{k\to\infty} S_k$.

If |r| < 1 then $\lim_{k\to\infty} r^{k+1} = 0$ and $\lim_{k\to\infty} S_k = \dfrac{a}{1-r}$ The geometric series **converges** to $a/(1-r)$.

If |r| > 1, we say the sum **diverges.** The sum also diverges if r = 1 or r = -1.

To summarize: If |r| < 1 then $a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^k + \cdots = a/(1-r)$

Find the sums of the 5 series given above, if they converge.

In sigma notation:

$a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^k + \cdots = \sum_{k=0}^{\infty} ar^k$ Note that this can also be written as

$\sum_{k=0}^{\infty} ar^k = \sum_{j=1}^{\infty} ar^{j-1} = \sum_{k=1}^{\infty} ar^{k-1}$ by changing the index variable.