Math 142 Supplementary Homework Problems

1. Differentiate and simplify: $y = \sin^2(3x)$
2. Differentiate and simplify: $y = \cos^{37}(2x)$
3. Differentiate and simplify: $y = \tan x \sec^3 x$
4. Differentiate and simplify: $y = \frac{1}{2}\ln(e^{2x} + 1)$

5. Differentiate and simplify: $y = \tan^{-1} x$

6. Find L_{10} and R_{10} for $f(x) = \frac{4}{1 + x^2}$ on [0, 1]. Also compute the average of these two estimates.

7. Differentiate and simplify: $y = \frac{x}{\sqrt{1 - x^2}}$

8. Differentiate and simplify: $y = x\cos x - \sin x$
9. Differentiate and simplify: $y = x\ln x - x$
10. Differentiate and simplify: $y = \ln(\ln x)$
11. Let f(x) = xe^x Find and simplify the first 4 derivatives of f(x). Then guess $\int xe^x \, dx$. Be sure to check your answer by differentiating.
12. Evaluate $\int_{-4}^{5} |x + 1| \, dx$ Hint: draw a picture
13. Let a = the last 4 digits of your social security number. Evaluate $\int_0^{a\pi} \sin x \, dx$

14. Differentiate and simplify: $y = \ln(e^{3x} + 1)$
15. Differentiate and simplify: $y = \ln\left(\frac{x - 1}{x + 1}\right)$
16. Differentiate and simplify: $y = e^{\sqrt{x}}$
17. Differentiate and simplify: $y = \tan^5 x$
18. Differentiate and simplify: $y = \sin^4(2x - 7)$

19. Find L_{10} and R_{10} for f(x) = 1/x on the interval [1, 11]. Compare these numbers with ln(11) (which is the area under the curve)

20. Evaluate $\int (x - 1)e^x \, dx$
21. Evaluate $\int \frac{e^{2x}}{e^{2x} + 1} \, dx$

22. Evaluate $\int 3\sin^4(3x-1)\cos(3x-1)\,dx$

23. Find the area above the x-axis and under y = tan x, for $0 \le x \le \pi/4$

24. The sequence of harmonic numbers H_n are defined by:

$H_n = \sum_{k=1}^{n} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ Hn = 1 + 1/2 + 1/3 + 1/4 + . . . + 1/n. The first few harmonic numbers are: $H_1 = 1$; $H_2 = 3/2$; $H_3 = 11/6$;

a) Find the smallest values of n that makes $H_n \ge 2$; $H_n \ge 3$; $H_n \ge 4$; (extra credit: find the smallest n for $H_n \ge 10$)

b) Show that $\ln(n+1) < H_n$ by comparing the area under y = 1/x with the harmonic numbers. (See the last homework)

25. Evaluate $\int_1^2 x^3 - 7x^2 + 1\,dx$

26. Evaluate $\int_{-4}^{-1} \frac{1}{x}\,dx$

27. Evaluate $\int \frac{e^x}{e^{2x}+1}\,dx$

29. Evaluate $\int \frac{1}{e^{2x}+1}\,dx$ hint: add to #21.

30. Evaluate $\int \tan x \sec^3 x\,dx$

31. Let $G(x) = \int_0^x \frac{1}{1+t+t^2}\,dt$. Find the intervals where G(x) is concave up.

32. What is the geometric series that corresponds to the periodic decimal x = .1212121212 . . . ?

(i) Use the formula for the sum of a geometric series to compute the value of x as a fraction.

(ii) What is the value of the periodic decimal y = .99999 . . . ?

33. Achilles and the tortise are having a race. The tortise can run 1 mile (or whatever the Hellenic equivalent of this would be) per hour. Achilles runs ten times as fast as the tortise so the tortise gets a head start of 1 mile. The race begins! By the time Achilles reaches the 1 mile mark, the tortise is .1 miles ahead. By the time Achilles runs this extra tenth of a mile, the tortise is still .01 miles ahead. This process continues; each time Achilles reaches the point where the tortise was, the tortise has moved ahead 1/10 as far. Can Achilles ever catch the tortise? If so, when? If not, who would you bet on?

34. What is the decimal expansion of 1/13? Is it periodic? (Do it using long division.) Does every rational number a/b have a periodic decimal expansion? Can you estimate the length of this period?

35. A superball is dropped vertically from a height of 10 feet. Each time it bounces, it reaches a height that's 75% of the height of the previous bounce. How far will the ball travel? Don't forget to count the distance up as well as the distance down,

36. If the ball in the previous problem keeps bouncing, and you could measure an arbitrarily small microscopic bounce, for how long would it bounce? Recall that the time it takes a ball to drop from a height of h ft is $t = h^{1/2} / 4$ seconds.

37. The formula for converting degrees Fahrenheit (x) to Celsius (f(x)) is f(x) = (5/9)(x - 32). What happens if you start with the last 4 digits of your student ID and keep converting it to Celsius? For example f(104) = 40; f(40) = 40/9 and f(40/9) = ? and so on. This is called iteration of a function.

38. Let $x_{n+1} = \sqrt{x_n + 1} \quad with\ x_0\ given$ Guess the value of $\lim_{x \to \infty} x_n$ if x0 = -.5 and also if x0 = 100.

38.5 Let $x_{n+1} = \sqrt{4x_n - 1} \quad with\ x_0\ given$ Guess the value of $\lim_{x \to \infty} x_n$ if $x_0 = 2 - \sqrt{3}$, $x_0 = .3$ and also if $x_0 = 0$.

39. The Fibonacci numbers $\{ F_n \}$ are defined by: $F_0 = 0$; $F_1 = 1$; $F_{n+1} = F_n + F_{n-1}$
The first few numbers are 0,1, 1, 2, 3, 5, 8, 13, 21, . . . To estimate how fast this sequence grows, compute the ratios F_{n+1}/ F_n for $10 < n < 20$

40. The sequence defined by $x_{n+1} = \sqrt{x_n + 1} \quad with\ x_0\ given$ has as a limit the solution to $x = \sqrt{x+1}\ or\ x^2 = x + 1$. Let the larger of these two solutions be denoted by ϕ, so $\phi = \frac{1+\sqrt{5}}{2}$. Then $\phi^2 = \phi + 1$, $\phi^3 = \phi^2 + \phi = \phi + 1 + \phi = 2\phi + 1$ and $\phi^4 = 2\phi^2 + \phi = 2(\phi + 1) + \phi = 3\phi + 2$. Compute the powers ϕ^n in terms of ϕ, for $5 < n < 10$. What is the pattern of the coefficients?

41. Apply the answer to the last problem to the other solution of $x^2 = x + 1$, denoted $\hat{\phi} = \frac{1-\sqrt{5}}{2}$. Subtract these two formulas for ϕ^n and $\hat{\phi}^n$ and solve for F_n.

42. Try to find a formula for these products of Fibonacci numbers $\{ F_{n-1} F_{n+1} \}$ for $n > 1$. To do this, write out a table containing n, F_n, and $F_{n-1} F_{n+1}$ for at least $0 < n < 10$ Then try to guess the pattern for the last column. (Hint: these numbers are almost squares.)
Here are the first few entries:

n	F_n	$F_{n-1} F_{n+1}$
0	0	-
1	1	0
2	1	2
3	2	3
4	3	10
5	5	24

43. Let R be the region bounded by $y = x^2$ and $y = x + 2$. Find:
a) the area of R
b) the volume of the solid if R is rotated about the x-axis
c) the volume of the solid if R is rotated about the the line x = 4

44. Let R be the region from the last problem. Find the center of mass of R. Assume that the density is constant.

45. Designer polynomials. Find a polynomial p(x) such that: p(0) = 0; p'(0) = 1; p''(0) = 2; and so on Make $p^{(k)}(0) = k$, for $0 \leq k \leq 7$.

46. Designer polynomials. Find a polynomial p(x) such that: p(0) = 0; p'(0) = 1; p''(0) = 1; p'''(0) = 2 and so on. Make $p^{(k)}(0) = (k-1)!$, for $0 \leq k \leq 7$.

47. Show $\lim_{y\to\infty} y^n e^{-y} = 0$ for any n > 0, then show $\lim_{x\to 0}\left(\frac{1}{x}\right)^k e^{-1/x^2} = 0$ for all k > 0.
Hint: Let $y = 1/x^2$.

48. Let $f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

Show that f(x) is continuous at x = 0, ie $\lim_{x\to 0} f(x) = f(0)$.

Show that f'(0) = 0 ie $\lim_{x\to 0}\frac{f(x)-f(0)}{x} = 0$

Show that f'(x) is also continous at x = 0. ie $\lim_{x\to 0} f'(x) = f'(0)$

49. Let $g(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

As in the previous problem, show that g is continuous at zero, determine g'(0) and show that g'(x) is continuous at x = 0.

50. Let $f(x) = xe^x$ Find the general pattern sequence of derivatives of f(x) at zero ie { $f^{(k)}(0)$ }. compare with problem #45.