Math 130 Worksheet for 2.7 and 2.8 - Answers

Let $f(x)=2x^4-20$ and $g(x)=\sqrt{x}$

1. Compute f∘ g (x) and determine its domain.

$f(g(x))=2(\sqrt{x})^4-20=2x^2-20$

The domain of f∘ g must be a subset of the domain of g, so the domain is $[0,\infty)$ This holds even though the domain of the composition could be extended to all x.

2. Compute g∘ f(x) and determine it domain. Is this the same as #1?

$g(f(x))=\sqrt{2x^4-20}$ The domain is all x for f, but its range must be restricted to the domain of g. This requires $2x^4-20\geq 0$ or $x^4\geq 10$ This gives a domain of $[-\infty,-10^{(1/4)})\cup[10^{(1/4)},\infty)$ As is usually the case, g∘ f and f∘ g are very different.

3. Decompose $h(x)=(3\sqrt{2x-4}+5)^4$ into simpler functions f and g so that h = f∘ g.

There are several ways to do this:

Let $f(u)=u^4$ *and* $g(x)=3\sqrt{2x-4}+5$ or $f(u)=(u+5)^4$ *and* $g(x)=3\sqrt{2x-4}$ or

$f(u)=(3u+5)^4$ *and* $g(x)=\sqrt{2x-4}$

4. $f(x)=\sqrt{3-x}$ What's f ∘ f (x)? f ∘ f ∘ f (x)? What happens if x = -1 and you keep applying f?

$f\circ f\circ f(x)=\sqrt{3-\sqrt{3-\sqrt{3-x}}}$

This is really a calculator exercise. f(-1) = 2; f(2) = 1; f(1) = $\sqrt{(2)}$;

To do continue the process, enter -1 into the calculator and press enter. The variable 'Ans' (on the TI calcs) now has the value -1. Then put in $\sqrt{(3-Ans)}$ and press enter several times. You should see { -1, 2, 1, 1.414, 1.259, . . . 1.30277563773 . . . }

If you think of f(x) as a machine which changes x to $\sqrt{(3-x)}$ then the limit of this sequences is a <u>fixed point</u> of f – one where f(x) = x.

Solve the equation $\sqrt{(3-x)}=x$ to determine the exact value of the limit.

Another example of this is convert a number from Fahrenheit to Celsius and keep on repeating the process: { 212, 100, 37.7, . . . } Where do you end up? Try converting in the opposite order. What happens?

5. Find the inverses of these functions, if possible.

a) f(x) = 2x – 5   y = 2x – 5 converts to x = (y + 5)/2; and so $f^{-1}(y) = (y + 5)/2$.

b) $g(x) = \dfrac{3x-2}{x+1}$ $y = \dfrac{3x-2}{x+1}$; $(x+1)y = 3x-2$; $xy - 3x = -y - 2$; $x = \dfrac{-y-2}{y-3} = \dfrac{y+2}{3-y}$

$g^{-1}(x) = \dfrac{x+2}{3-x}$ Note domain of g = {x ≠ 1} and range of g = domain of g^{-1} = {x ≠ 3}

c) $h(x) = 2x^2 - 20$ Solving for x gives $y = 2x^2 - 20$; $x^2 = \dfrac{y+20}{2}$; $x = \pm\sqrt{y/2 + 10}$

The $\pm$ indicates that h has no inverse. Draw a graph of h – it's a parabola – a function that fails the horizontal line test – there is more than one y value for each nonzero x.

In parts d and e there are inverses.

d) h domain = $\left(-\infty, -\sqrt{(10)}\right]$ means that x should be negative: $h^{-1}(y) = -\sqrt{y/2 + 10}$

e) h domain = $\left[\sqrt{(10)}. \infty\right)$ means that x should be positive: $h^{-1}(y) = \sqrt{y/2 + 10}$

In both cases the domain of h^{-1} is $[0, \infty]$ since this is the restricted range of h(x). The h domains in d and e can be extended to $(-\infty, 0]$ and $[0, \infty)$ and they will still have the same inverse functions.

6. Sketch a graph of the inverse of y = f(x), given its graph below.