Math 142: Review Sheet for Fourth Test

5.9 Approximate Integration
   (a) Midpoint rule
   (b) Trapezoid Rule
   (c) Simpson's Rule
   (d) Error estimation

5.10 Improper integrals
   (a) Calculating using limits
   (b) Comparison tests

6.1 Area between curves
   (a) Area between the graphs of functions
   (b) Area under parametric curves

6.2 Volumes
   (a) Disk/washer method
   (b) Cylindrical shells

6.3 Arc length
   (a) Arc length of parametric curves
   (b) Arc length for graphs of functions

Practice Problems:

(1) Estimate $\int_2^3 \sin(\cos x)\,dx$ using

  (a) the midpoint rule with $n = 6$.

  (b) the trapezoid rule with $n = 6$.

  (c) Simpson's rule with $n = 6$.

  Note: make sure your calculator is in radians mode!

(2) Show that Simpson's rule is exact for $\int_a^b x^3\,dx$. Hint: use the error bound formula on page 419.

(3) How large do we need to choose n so that we calculate $\int_0^1 x^4\,dx$ to within .0001, using

  (a) the midpoint rule

  (b) Simpson's rule

(4) Draw the graphs of a function on the interval $[0, 1]$, where the trapezoid rule is much more accurate than simpson's rule.

(5) Calculate $\int_{-3}^{5} \frac{1}{\sqrt[3]{x+3}}\,dx$.

(6) Is there a value of p for which $\int_0^\infty \frac{1}{x^p}\,dx$ converges?

(7) Calculate $\int_0^\infty xe^{-x}\,dx$.

(8) Calculate $\int_1^\infty x^{-5}\,dx$.

(9) Use the comparison test to determine whether the following integrals converge or diverge.

  (a) $\int_0^{\pi/2} \frac{dx}{x \sin x}$

  (b) $\int_0^1 \frac{e^{-x}}{\sqrt{x}}\,dx$.

(10) Find the values of p for which the integral

$$\int_e^\infty \frac{1}{x(\ln x)^p}\,dx$$

converges.

(11) Sketch the region enclosed by the given curves and then find the area of the region.

(a) $y = 1/x$, $y = x$, $y = \frac{1}{4}x$, $x > 0$.

(b) $y = |x|$, $y = x^2 - 2$.

(12) For what values of m do the line $y = mx$ and the curve $y = \dfrac{x}{x^2+1}$ enclose a region? Find the area of the region.

(13) Find an integral describing the volume of a right circular cone of height h and base radius r.

(14) The region enclosed by the curves $y = x^3$ and $y = \sqrt{x}$ is rotated about the line $y = 1$. Find the volume of the resulting solid.

(15) Find the volume of a sphere of radius 2 with a cylindrical tunnel of radius 1 drilled through it.

(16) Find the arc length of the curve $x = t^2 + 1, y = 3t^3$ from $t = 0$ to $t = 2$.

(17) Find the length of the graph of the function $y = -\ln(\cos x)$ from $x = 0$ to $x = \pi/6$.

Answers:

(1) Midpoint: $-.685848$, Trapezoid: $-.683226$, Simpson: $-.684979$.

(2) We can take $K = 0$ in the error formula.

(3) Midpoint: $n \geq 70.7 \Rightarrow n \geq 71$. Simpsons: $n \geq 6.04275 \Rightarrow n \geq 8$. (Remember n must be even for Simpson's rule.)

(4)

(5) 6

(6) No.

(7) 1

(8) 1/4

(9) a) diverges b) converges

(10) $p > 1$.

(11) (a) $\int_0^1 (x - x/4)\, dx + \int_1^2 (1/x - x/4)\, dx = \ln 2$.

(b) $2\int_0^2 (x - (x^2 - 2))\, dx = 10/3$

(12) $0 < m < 1$. Area $= \displaystyle\int_0^{\sqrt{\frac{1}{m}-1}} \left(\frac{x}{x^2+1} - mx \right) dx = \frac{1}{2}(m - 1 - \ln m)$

(13) One possible answer is $\int_0^h \pi(r - (r/h)y)^2\, dy$.

(14) $\int_0^1 \pi(1 - x^3)^2 - \pi(1 - \sqrt{x})^2\, dx = 10\pi/21$.

(15) $\displaystyle\int_{-\sqrt{3}}^{\sqrt{3}} \pi(\sqrt{4 - y^2})^2 - \pi(1)^2\, dy = 4\sqrt{3}\pi$.

(16) $\int_0^2 \sqrt{(2t)^2 + (9t^2)^2}\, dt = \frac{8}{243}(-1 + 82\sqrt{82})$

(17) $\int_0^\pi /6\sqrt{1 + \tan^2 x}\, dx = \ln 3/2$.