Have a warehouse with no heating/AC and a time constant of 4. Suppose the outside temp on an average day hits a high of 100°F (about 4pm) and a low of 60°F (about 4am), and that at 4am one morning the temp was 65°F inside the warehouse. What is the hottest and coldest temp that is reached in the warehouse over the next few days?

Model: $M(t) = \text{outside temp} = 80 + 20\sin\left(\frac{\pi t}{12} - \frac{\pi}{2}\right)$

so

$$\frac{dT}{dt} = \frac{1}{4}\left(80 + 20\sin\left(\frac{\pi t}{12} - \frac{\pi}{2}\right) - T\right)$$

$$\Rightarrow \frac{dT}{dt} + \frac{1}{4}T = 20 + 5\sin\left(\frac{\pi t}{12} - \frac{\pi}{2}\right) \quad \text{(linear!)}$$

$$\Rightarrow \frac{d}{dt}\left(e^{\frac{t}{4}}T\right) = 20e^{t/4} + 5e^{t/4}\sin\left(\frac{\pi t}{12} - \frac{\pi}{2}\right)$$

$$\Rightarrow e^{\frac{t}{4}}T = 80e^{t/4} + 5\int e^{t/4}\sin\left(\frac{\pi t}{12} - \frac{\pi}{2}\right)dt$$

(* see last page)

$$= 80e^{t/4} + 5\left[\frac{4e^{t/4}\sin\left(\frac{\pi t}{12} - \frac{\pi}{2}\right) - \frac{16\pi}{12}e^{t/4}\cos\left(\frac{\pi t}{12} - \frac{\pi}{2}\right)}{\left(1 + \frac{16\pi^2}{(12)^2}\right)}\right] + C$$

gen. sol'n:

$$\Rightarrow \boxed{T(t) = 80 + \left(\frac{5 \cdot 144}{144 + 16\pi^2}\right)\left[4\sin\left(\frac{\pi t}{12} - \frac{\pi}{2}\right) - \frac{16\pi}{12}\cos\left(\frac{\pi t}{12} - \frac{\pi}{2}\right)\right] + Ce^{-\frac{t}{4}}}$$

if $t=0$ is 4am $\Rightarrow T(0)=65 \Rightarrow$

$$-15 = \left(\frac{5\cdot 144}{144+16\pi^2}\right)(-4) + C$$

$$\Rightarrow \boxed{C = \frac{20\cdot 144}{144+16\pi^2} - 15 \approx -5.46}$$

Max possible temperature? min?

Find critical points of T!

$$T'(t) = \frac{\pi}{12}\left(\frac{5\cdot 144}{144+16\pi^2}\right)\left[4\cos\left(\frac{\pi t}{12}-\frac{\pi}{2}\right) + \frac{16\pi}{12}\sin\left(\frac{\pi t}{12}-\frac{\pi}{2}\right)\right] - \frac{1}{4}Ce^{-t/4} = 0$$

assume t is large enough that $e^{-t/4}$ is negligible

$$T'=0 \quad \Rightarrow \quad \tan\left(\frac{\pi t}{12}-\frac{\pi}{2}\right) = \frac{-48}{16\pi} = \frac{-3}{\pi}$$

apply arctan to both sides $\Rightarrow \quad \frac{\pi t}{12} - \frac{\pi}{2} \approx -0.7623 + n\pi$ for any integer n,

$$\Rightarrow \boxed{t \approx 3.088 + 12n \approx 3 + 12n}$$

says extreme temps will occur around $t = 3, 15, 27,$ etc...

$$T(\underset{7am}{3}) = 80 + \left(\frac{5\cdot 144}{144+16\pi^2}\right)\left[4\left(-\frac{\sqrt{2}}{2}\right) - \frac{16\pi}{12}\left(\frac{\sqrt{2}}{2}\right)\right] \approx 63.43 \text{ (min)}$$

[redacted]

$$T(\underset{17pm}{15}) = 80 + \left(\frac{5\cdot 144}{144+16\pi^2}\right)\left[4\left(\frac{\sqrt{2}}{2}\right) + \frac{16\pi}{12}\left(\frac{\sqrt{2}}{2}\right)\right] \approx 93.09 \text{ (max)}$$

Find:

$$\boxed{\int e^{at}\sin(bt+c)\,dt} = \frac{1}{a}e^{at}\sin(bt+c) - \int \frac{b}{a}e^{at}\cos(bt+c)\,d$$

$$\boxed{\begin{array}{ll} u = \sin(bt+c) & du = b\cos(bt+c)\,dt \\ dv = e^{at}\,dt \quad \Rightarrow & v = \frac{1}{a}e^{at} \end{array}}$$

$$= \frac{1}{a}e^{at}\sin(bt+c) - \frac{b}{a}\left[\frac{1}{a}e^{at}\cos(bt+c) + \frac{b}{a}\int e^{at}\sin(bt\right.$$

$$\boxed{\begin{array}{ll} u = \cos(bt+c) & du = -b\sin(bt+c)\,dt \\ dv = e^{at}\,dt \quad \Rightarrow & v = \frac{1}{a}e^{at} \end{array}}$$

$$\Rightarrow \int e^{at}\sin(bt+c)\,dt = \frac{1}{a}e^{at}\sin(bt+c) - \frac{b}{a^2}e^{at}\cos(bt+c) - \underbrace{\frac{b^2}{a^2}\int e^{at}\sin(bt+c)\,dt}_{(*)}$$

adding (*) to both sides gives:

$$\left(1+\frac{b^2}{a^2}\right)\int e^{at}\sin(bt+c)\,dt = \frac{1}{a}e^{at}\sin(bt+c) - \frac{b}{a^2}e^{at}\cos(bt+c)$$

$$\Rightarrow \boxed{\int e^{at}\sin(bt+c)\,dt = \frac{\frac{1}{a}e^{at}\sin(bt+c) - \frac{b}{a^2}e^{at}\cos(bt+c)}{\left(1+\frac{b^2}{a^2}\right)} + D}$$

arbitrary constant.