Solutions

Work-It-Out Day 4 Supplemental Problems

1. $\int_2^4 \frac{1}{3t+4}\,dx$

$u = 3t+4 \Rightarrow u(2) = 6\ ,\ u(4) = 16$

$du = 3dt$

$\frac{1}{3}du = dt$

$$\frac{1}{3}\int_6^{16} \frac{1}{u}\,du$$

$$= \frac{1}{3}\ln|u|\Big|_6^{16} = \frac{1}{3}\ln|16| - \frac{1}{3}\ln|6|$$

2. $\int \frac{x^2}{2x+3}\,dx$

let: $u = 2x+3 \Rightarrow x = \frac{u-3}{2}$

$du = 2dx$

$\frac{1}{2}du = dx$

$$\int \frac{x^2}{2x+3}\,dx = \frac{1}{2}\int \frac{\left(\frac{u-3}{2}\right)^2}{u}\,du$$

$$= \frac{1}{8}\int \frac{(u-3)^2}{u}\,du = \frac{1}{8}\int \frac{u^2-6u+9}{u}\,du$$

$$= \frac{1}{8}\left[\int u - 6 + \frac{9}{u}\,du\right] = \frac{1}{8}\left[\frac{1}{2}(2x+3)^2 - 6(2x+3) + 9\ln|2x+3|\right] + C$$

3. $\int x^7(x^4+2)^{4/5}\,dx$

let: $u = x^4+2 \Rightarrow x^4 = u-2$

$\Rightarrow du = 4x^3dx$

$\Rightarrow \frac{1}{4}du = x^3dx$

$$\int x^7(x^4+2)^{4/5}du = \int x^4 (x^4+2)^{4/5} x^3\,dx$$

$$= \frac{1}{4}\int (u-2)u^{4/5}\,du = \frac{1}{4}\int u^{9/5} - 2u^{4/5}\,du$$

$$= \frac{1}{4}\left[\frac{5u^{14/5}}{14} - \frac{2\cdot 5}{9}u^{9/5}\right] + C = \frac{1}{4}\left[\frac{5}{14}(x^4+2)^{14/5} - \frac{10}{9}(x^4+2)^{9/5}\right] + C$$

let $u = 1-x^2$

$du = -2x\,dx$

$-2du = 4x\,dx$

4. $\int_{-1/2}^{0} \frac{4x+1}{\sqrt{(1-x^2)}}\,dx = \int_{-1/2}^{0} \frac{4x}{\sqrt{1-x^2}} + \int_{-1/2}^{0} \frac{1}{\sqrt{1-x^2}}\,dx$

$$= -2\int_{3/4}^{1} \frac{1}{\sqrt{u}}\,du + \left(\arcsin(x)\Big|_{-1/2}^{0}\right) = -2\left(\frac{u^{1/2}}{1/2}\right)\Big|_{3/4}^{1} + \arcsin(x)\Big|_{-1/2}^{0}$$

$$= -4\left(\sqrt{1} - \sqrt{3/4}\right) + \arcsin(0) - \arcsin(-1/2)$$

5. $\int \frac{3}{4x^2+10}\,dx$

want: $4x^2 = 10u^2 \Rightarrow 2x = \sqrt{10}\,u$

or $u = \frac{2}{\sqrt{10}}x \Rightarrow du = \frac{2}{\sqrt{10}}\,dx$

$\Rightarrow \frac{\sqrt{10}}{2}\,du = dx$

$$= 3\int \frac{1}{4x^2+10}\,dx$$

$$= 3\int \frac{1}{10u^2+10} \cdot \frac{\sqrt{10}}{2}\,du = \frac{\sqrt{10}\cdot 3}{2\cdot 10}\int \frac{1}{u^2+1}\,du$$

$$= \frac{3}{2\sqrt{10}}\arctan(u) + C = \frac{3}{2\sqrt{10}}\arctan\left(\frac{2x}{\sqrt{10}}\right) + C$$

6. $\int \tan(x)(\ln(\cos(x)))^5\,dx$

method 1:

$$= \int \frac{\sin(x)(\ln(\cos(x)))^5}{\cos(x)}\,dx$$

$u = \cos(x)$

$du = -\sin(x)dx$

$w = \ln(u)$

$dw = \frac{1}{u}du$

$$= -\int \frac{(\ln(u))^5}{u}\,du$$

$$= -\int w^5\,dw = -\frac{1}{6}w^6 + C$$

$$= -\frac{1}{6}(\ln(\cos(x)))^6 + C$$

method 2:

$u = \ln(\cos(x))$

$du = \frac{1}{\cos(x)} \cdot -\sin(x)dx = -\tan(x)dx$

$$\Rightarrow = \int -u^5\,du$$

$$= -\frac{1}{6}u^6 + C$$

$$= -\frac{1}{6}(\ln(\cos(x)))^6 + C$$