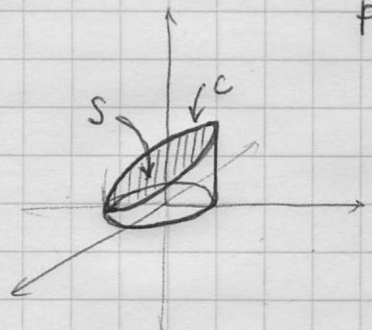
## 4.9 – #10

Use Stoke's thm to find $\oint_C \vec{F} \cdot d\vec{R}$

around the ellipse $x^2+y^2=1$, $z=y$ where

$\vec{F}(x,y,z) = \langle x, x+y, x+y+z \rangle$




Let C be the ellipse, S be the surface (disk) bounded by the curve C.

By Stoke's theorem $\oint_{C=\partial S} \vec{F} \cdot d\vec{R} = \iint_S \vec{\nabla} \times \vec{F} \cdot d\vec{S}$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ x & x+y & x+z+y \end{vmatrix} = \langle 1, -1, 1 \rangle$$

we can parametrize S by noticing it lies within the plane $z=y$ and is a disk:

$$\vec{R}(r,\theta) = \langle r\cos\theta, r\sin\theta, r\sin\theta \rangle$$
$$0 \le r \le 1, \quad 0 \le \theta < 2\pi$$

$$\Rightarrow d\vec{S} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos\theta & \sin\theta & \sin\theta \\ -r\sin\theta & r\cos\theta & r\cos\theta \end{vmatrix} = 0\vec{i} + -r\vec{j} + r\vec{k} = \langle 0, -r, r \rangle \, dr\, d\theta$$

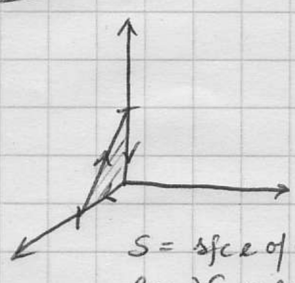
$$\Rightarrow \iint_S \vec{\nabla}\times\vec{F} \cdot d\vec{S} = \int_0^{2\pi}\int_0^1 \langle 1,-1,1 \rangle \cdot \langle 0,-r,r \rangle \, dr\, d\theta = \int_0^{2\pi}\int_0^1 2r\, dr\, d\theta = \int_0^{2\pi} \frac{1}{2}\, d\theta = \pi$$

## 4.9 – #16:

$\vec{F} = \langle xz, -y, x^2y \rangle$

Evaluate $\oint_C \vec{F} \cdot d\vec{R}$ by Stoke's theorem

C is the path along straight edges from (1,0,0) to (0,0,1) to (0,0,0) and back to (1,0,0).



S = sfce of triangle
$C = \partial S$ = edges of triangle

$$\oint_C \vec{F} \cdot d\vec{R} \underset{\text{Stokes'}}{=} \iint_S \vec{\nabla}\times\vec{F} \cdot d\vec{S}$$

The surface S is parametrized by $y=0$, $0 \le x \le 1$, $0 \le z \le 1-x$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial_x & \partial_y & \partial_z \\ xz & -y & x^2y \end{vmatrix} = \vec{i}(x^2) + (x - 2xy)\vec{j} + 0\vec{k}$$
$$= \langle x^2, x-2xy, 0 \rangle$$

$$= \int_0^1\int_0^{1-x} \langle x^2, x, 0 \rangle \cdot \langle 0, -1, 0 \rangle \, dz\, dx$$

$$= \int_0^1\int_0^{1-x} -x\, dz\, dx = \int_0^1 -x(1-x)\, dx = -\frac{1}{2}x^2 + \frac{1}{3}x^3 \Big|_0^1$$

$$= -\frac{1}{2} + \frac{1}{3} = -\frac{1}{6}$$